Special Cases of Quadratic Equations

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Recall: We have learned that a TRINOMIAL quadratic equation can be solved by

1- Factoring

2- Using the Quadratic Formula

$$ax^2+bx+c=0$$

What if the quadratic was not a trinomial?

Ex.1. Solve.

a) $x^2 + 2x = 0$

$x(x+2)=0$

$x=0$ or $x+2=0$

$x=-2$

b) $x^2 - 36 = 0$

$(x+6)(x-6)=0$

$x+6=0$ or $x-6=0$

$x=-6$ $x=6$

Taking the Square Root of a Variable

Evaluate the following:

(a) $\sqrt{4} = 2$ (b) $\sqrt{81} = 9$ (c) $\sqrt{17} \doteq 4.123$

Now consider:

(a) $\sqrt{x^2} = \pm x$ (b) $\sqrt{9y^2} = \pm 3y$

Reconsider $x^2 - 36 = 0$

$x^2 = 36$

$\sqrt{x^2} = \pm\sqrt{36}$

$x = \pm 6$

$\pm x = 6$

$x = 6$ or $-x = 6$

$x = -6$

Ex.2. Solve.

a) $4x^2 - 5 = 0$

$4x^2 = 5$

$x^2 = \frac{5}{4}$

$x = \pm\sqrt{\frac{5}{4}}$

$x \doteq \pm 1.118$

b) $2x^2 + 16 = 0$

$2x^2 = -16$

$x^2 = -8$

$x = \pm\sqrt{-8}$

does not exist (DNE)

c) $2(x + 3)^2 - 7 = 0$

$2(x+3)^2 = 7$

$(x+3)^2 = \frac{7}{2}$

$x+3 = \pm\sqrt{\frac{7}{2}}$

$x = -3 \pm \sqrt{\frac{7}{2}}$

$x = -3+\sqrt{\frac{7}{2}}$ or $x = -3-\sqrt{\frac{7}{2}}$

$x \doteq -1.129$ $\doteq -4.87$

Summary:

Below are some strategies to help us solve quadratic equations more efficiently.

1. Trinomial → Factor / Quadratic Formula $ax^2+bx+c=0$

2. Binomial with linear term → Common Factor $x^2+2x=0$

3. Binomial with constant term → Isolate x^2 $x^2-36=0$
   (If there is a subtraction between the two terms it can be factored as a difference of two squares.)

4. $(x \pm h)^2$ term → Isolate $(...)^2$

Assigned Work

p. 343 # 3, 4, 5cd

On a blank piece of paper, make up the following 4 types of quad equations and solve.

1- Binomial equation with linear term: $ax^2 \pm bx = 0$
2- Binomial equation with constant term: $ax^2 \pm c = 0$
3- Equation: $a(x - h)^2 + k = 0$
4- Trinomial: $ax^2 \pm bx \pm c = 0$

p. 343
3(d)

$x(x-2) = 36 - 2x$

$x^2 - 2x = 36 - 2x$

$+2x \quad -36 \quad +2x$

-36

$x^2 - 36 = 0$

$(x-6)(x+6) = 0$

$x = 6$ or $x = -6$

$x^2 = 36$

$x = \pm 6$

4(a) $(x+1)^2 - 16 = 0$

$(x+1)^2 = 16$

$x+1 = \pm 4$

$x = -1 \pm 4$

$x = -5$ or $x = 3$

4(b) $-2(x+5)^2 + 2 = 0$

$\frac{-2(x+5)^2}{-2} = \frac{-2}{-2}$

$(x+5)^2 = 1$

$x+5 = \pm 1$

$x = -5 \pm 1$

$x = -4$ or $x = -6$

5(c) $x^2 - 16 = 0$ $\quad x^2 + 0x - 16 = 0$

$a = 1 \quad c = -16$

$b = 0$

$(x-4)(x+4) = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(0) \pm \sqrt{(0)^2 - 4(1)(-16)}}{2(1)}$$

$$= \frac{\pm \sqrt{64}}{2}$$

$$= \frac{\pm 8}{2}$$

$x = 4$ OR $x = -4$