

Triangle Centres

march 7/2011

There is more than one centre for triangles.

The **centroid** is the intersection point of the **medians**.

The **orthocentre** is the intersection point of the **altitudes**.

The **circumcentre** is the intersection point of the **perpendicular bisectors**.

The **incentre** is the intersection point of the **angle bisectors**. *** will not be evaluated ***

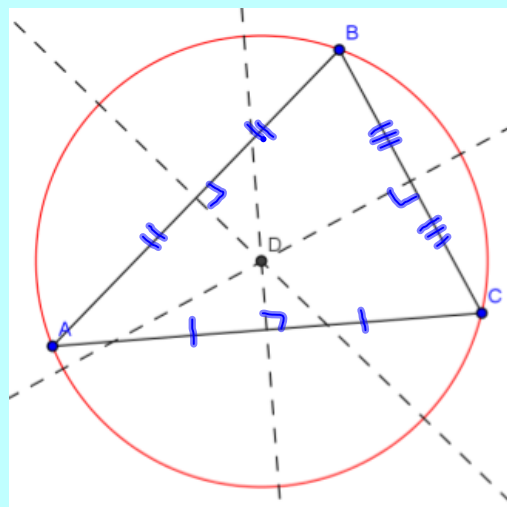
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The **centroid** is also known as the centre of mass of the triangle. You could balance the triangle at this point.

The **circumcentre** is the point that is equidistant from all 3 vertices of the triangle.

or

It the centre of the circle that passes through each vertex of the triangle.



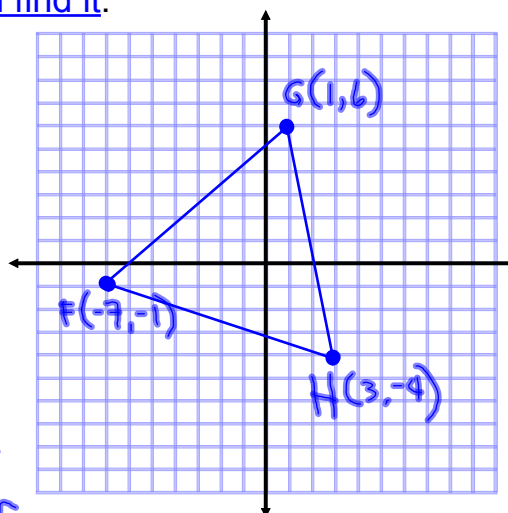
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Given triangle FGH with vertices at F(-7,-1), G(1,6), and H(3,-4):

a) List the steps required to determine the coordinates of the circumcentre, and then find it.

(draw a sketch first!)

- ① find the midpoints of two sides
- ② find slopes of same two sides
- ③ find M_{\perp} for 2 sides
- ④ find equation of each perpendicular bisector
- ⑤ solve two equations for circumcentre
- ⑥ find 3rd line and verify point



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Given triangle FGH with vertices at F(-7,-1), G(1,6), and H(3,-4):

a) List the steps required to determine the coordinates of the circumcentre, and then find it.
(draw a sketch first!)

① $M_{FG} = \left(\frac{-7+1}{2}, \frac{-1+6}{2} \right)$
 $= \left(-3, \frac{5}{2} \right)$
 $M_{FH} = \left(\frac{-7+3}{2}, \frac{-1+(-4)}{2} \right)$
 $= \left(-2, -\frac{5}{2} \right)$

② $m_{FG} = \frac{6-(-1)}{1-(-7)} = \frac{7}{8}$
 $m_{FH} = \frac{-4-(-1)}{3-(-7)} = \frac{-3}{10}$
 $m_{\perp FG} = -\frac{8}{7}$
 $m_{\perp FH} = \frac{10}{3}$

④ $l_1: y = -\frac{8}{7}x + b$ $l_2: y = \frac{10}{3}x + b$
 Sub $\left(-3, \frac{5}{2} \right)$ Sub $\left(-2, -\frac{5}{2} \right)$
 $\frac{5}{2} = -\frac{8}{7}(-3) + b$ $-\frac{5}{2} = \frac{10}{3}(-2) + b$
 $\frac{5}{2} = \frac{24}{7} + b$ $-\frac{5}{2} = -\frac{20}{3} + b$
 $\frac{5}{2} - \frac{24}{7} = b$ $-\frac{5}{2} + \frac{20}{3} = b$
 $\frac{25}{14} - \frac{24}{7} = b$ $-\frac{15}{6} + \frac{40}{6} = b$
 $\frac{25}{14} - \frac{48}{14} = b$ $\frac{25}{6} = b$
 $-\frac{23}{14} = b$ $\frac{25}{6} = b$

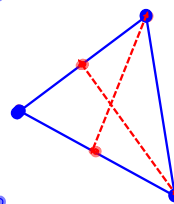
$l_1: y = -\frac{8}{7}x - \frac{23}{14}$ $l_2: y = \frac{10}{3}x + \frac{25}{6}$

to solve: $y = y$
 $-\frac{8}{7}x - \frac{23}{14} = \frac{10}{3}x + \frac{25}{6}$
 $-\frac{24}{21} - \frac{25}{14} = \frac{10}{3}x + \frac{25}{6}$
 $-\frac{32}{42} - \frac{175}{42} = \frac{20}{21}x + \frac{25}{6}$
 $-\frac{207}{42} = \frac{20}{21}x + \frac{25}{6}$
 $-\frac{207}{42} - \frac{25}{6} = \frac{20}{21}x$
 $-\frac{207}{42} - \frac{175}{42} = \frac{20}{21}x$
 $-\frac{382}{42} = \frac{20}{21}x$
 $-\frac{191}{21} = \frac{20}{21}x$
 $-\frac{191}{20} = x$
 $-\frac{191}{20} = x \rightarrow y = \frac{10}{3}x + \frac{25}{6}$
 $y = \frac{10}{3} \left(-\frac{191}{20} \right) + \frac{25}{6}$
 $y = -\frac{191}{6} + \frac{25}{6}$
 $y = -\frac{166}{6}$
 $y = -\frac{83}{3}$
 $\therefore \text{circumcentre } (-\frac{191}{20}, -\frac{83}{3})$

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b) List the steps required to determine the coordinates of the centroid.

→ intersection of medians
midpoint to vertex. ↙



- ① find midpoints of 2 sides
- ② use vertex opposite each side and MP to find

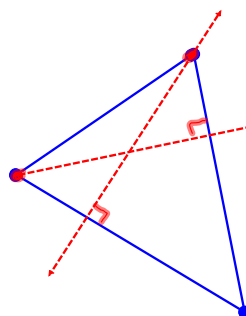
(a) slope (b) y-int	}	equation of each median
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- ③ Solve system of equations (medians) for the centroid.

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c) List the steps required to determine the coordinates of the orthocentre.

intersection of altitudes

↙
perpendicular lines passing through each vertex



- ① find slopes of 2 sides
- ② neg. reciprocals → slopes of altitudes
- ③ y-int of altitudes using vertex as point → equation of altitudes
- ④ Solve system for orthocentre.

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Assigned Work:

p.120-121 # 6, 8, 9, 10

Triangle ABC has vertices A(3, 4), B(-5, 2) and C(1, -4).

Find the coordinates of the

a) circumcentre. Answer: $(-2/5, 3/5)$

b) orthocentre. Answer: $(-1/5, 4/5)$

c) centre of mass (centroid). Answer: $(-1/3, 2/3)$

Test will be Friday:

Review:

p124-125 #1, 2, 3, 6, 7, 8, 9, 10, 11, 13,
15, 16, 18, 20a, 21, 22, 23

Feb 28-12:00 PM