Relating Three Forms of a Quadratic using Factoring

Apr. 15/2011

Recall:

Standard Form $y = ax^2 + bx + c$

Factored Form $y = a(x - s)(x - t)$

Vertex Form $y = a(x - h)^2 + k$

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Last unit we worked on expanding, simplifying, and factoring. These are the skills to go between the standard and the factored form.

Ex: $y = x^2 - 4x + 3$ is **factored** to obtain the factored form

$y = (x - 1)(x - 3)$

Ex: $y = 2(x + 5) (x - 1)$ is **expanded and simplified** to obtain the standard form

$y = 2(x^2 - x + 5x - 5)$

$y = 2(x^2 + 4x - 5)$

$y = 2x^2 + 8x - 10$

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Ex: $y = -(x+3)^2 - 4$ is **expanded and simplified** to obtain the standard form

$y = -(x+3)(x+3) - 4$

$y = -(x^2+3x+3x+9) - 4$

$y = -1(x^2+6x+9) - 4$

$y = -x^2 - 6x - 9 - 4$

$y = -x^2 - 6x - 13$

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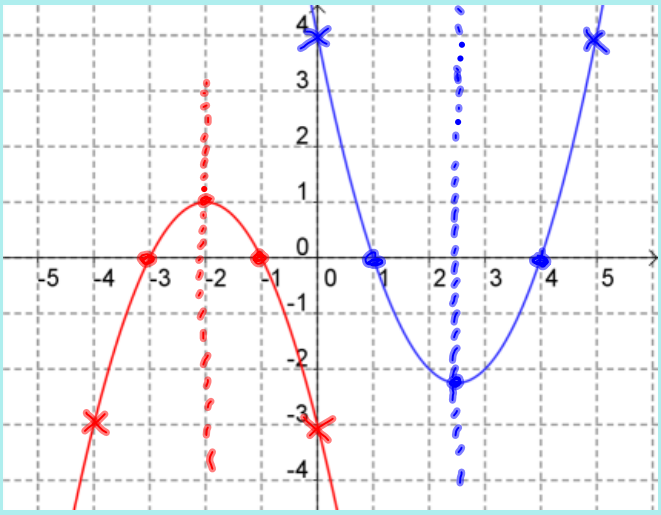
Obtaining the vertex form requires a new process!



Earlier in the unit we looked at the properties of parabolas and we discovered that the vertex is always between of the zeros or of any other two points that are at the same height.

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Method 1: Determine the vertex (h, k) from the zeros.

x y

1) Determine the zeros.

2) Find the *x*-value of the vertex (h),

h = midpoint of the zeros

$$= \frac{x_1 + x_2}{2}$$

$$= \frac{s + t}{2}$$

from $y = a(x-s)(x-t)$

3) Find the *y*-value of the vertex (k),

Sub x-value found for h into original equation

4) To write in vertex form, recall that the a-term is the same in all forms.

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Ex: Determine the vertex, and the vertex form, of
$y = 2(x - 3)(x + 5)$

① for zeroes, set $y = 0$

$0 = 2(x-3)(x+5)$

$x-3=0$ or $x+5=0$

$x=3$ $\quad x=-5$

② $x_m = \frac{3+(-5)}{2}$

$= \frac{-2}{2}$

$= -1 \longrightarrow h = -1$

③ sub $x=-1$ into $y=2(x-3)(x+5)$

$y = 2(-1-3)(-1+5)$

$y = 2(-4)(4)$

$y = -32 \longrightarrow k = -32$

Vertex is $(-1,-32)$ and $a=2$

$y = a(x-h)^2 + k$

$y = 2(x-(-1))^2 + (-32)$

$y = 2(x+1)^2 - 32$

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Ex: Determine the vertex, and the vertex form, of
$y = 5x^2 - 15x - 20$

GCF = 5

$y = 5(x^2 - 3x - 4)$

$y = 5(x - 4)(x + 1)$

follows steps 1-4 to find vertex form.

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Ex: Determine the vertex, and the vertex form, of
$y = x^2 - 12x + 5$

cannot be factored

y-int

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The problem with this method? Not all quadratics have zeros!

Method 2: Determine the vertex (h, k) from any two points that have the same *y*-value.

1) Determine two points that have the same y-value.
partial factoring is most effective here:
factor the first two terms of the standard form
set each partial factor equal to zero

2) Find the *x*-value of the vertex (h),
h = midpoint of the *x*-values of the two points

$$= \frac{x_1 + x_2}{2}$$

3) Find the *y*-value of the vertex (k),
Substitute x-value found for h into original equation

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Ex: Determine the vertex, and the vertex form, of
$y = x^2 - 12x + 5$

cannot be factored, use partial factoring

y-int = 5 → point (0,5)

look for another point where y=5

Set y=5, find x

$5 = x^2 - 12x + 5$

$-5 \quad\quad -5$

$0 = x^2 - 12x$

$0 = x(x-12)$ $\quad a \cdot b = 0$

$x = 0$ or $x - 12 = 0$, $x = 12$

$x_m = \frac{0+12}{2}$

$= 6$

Sub $x = 6$ into $y = x^2 - 12x + 5$

$y = (6)^2 - 12(6) + 5$

$= 36 - 72 + 5$

$y = -31$

Vertex $(6, -31)$

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Ex: Determine the vertex, and the vertex form, of
$y = -3x^2 + 15x + 2$

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Assigned Work:

p.301 # 1, 5, 7

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