Applying our knowledge of Quadratics to Word Problems

Apr 27/2011

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What we have learned that we will be using:

- factoring and the quadratic formula leads to the roots

- finding the vertex (by factoring, partial factoring, or completing the square) gives you the optimal value

Remember that in word problems it is always important to identify the variables and sketching the parabola can be useful

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Applying our knowledge of Quadratics to Word Problems

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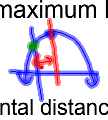
Ex. 1) A hose is placed on an aerial ladder. The hose sprays water on a forest fire. The height of the water, *h*, in metres can be modelled by the relation

$$h = -2.25(d - 1)^2 + 9,$$

where *d* is the horizontal distance, in metres, of the water from the nozzle of the hose.

a) What is the maximum height reached by the water?

$V(1, 9)$
opens down



$\therefore$ max height is 9m
(optimal value)

b) At what horizontal distance from the nozzle is the max height reached?

max height is 1m horizontally from nozzle

c) What is the height of the aerial ladder?

ladder + hose at y-int
to find y-int, set x = 0

$h = -2.25(0-1)^2 + 9$
$= -2.25 + 9$
$= 6.75$

$\therefore$ the ladder is 6.75m high

d) How high is the water when it is at a horizontal distance of 2m from the nozzle?

set $d = 2$
$h = -2.25(2-1)^2 + 9$
$= -2.25 + 9$
$= 6.75$

$\therefore$ the water is 6.75m high at d = 2m.

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Ex. 2) A ball is thrown into the air. Its height, in metres, after *t* seconds is $h = -4.9t^2 + 39.2t + 1.75$.

a) When does it reach maximum height?

$h = -4.9t^2 + 39.2t + 1.75$
$= -4.9(t^2 - 8t) + 1.75$
$= -4.9(t^2 - 8t + 16 - 16) + 1.75$
$= -4.9[(t-4)^2 - 16] + 1.75$
$= -4.9(t-4)^2 + 78.4 + 1.75$
$= -4.9(t-4)^2 + 80.15$

$\frac{-8}{2} = -4$
$(-4)^2 = 16$

$\therefore$ it reaches the max. height after 4 seconds

b) What is the maximum height?

$\therefore$ the max. height is 80.15m

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Ex. 2) continued...

A ball is thrown into the air. Its height, in metres, after t seconds is $h = -4.9t^2 + 39.2t + 1.75$.

c) From what height is the ball released?

initial height occurs at $t = 0$ (same as $x=0$ → y-int)

set $t=0$, $h = 1.75$

∴ the initial height is 1.75m

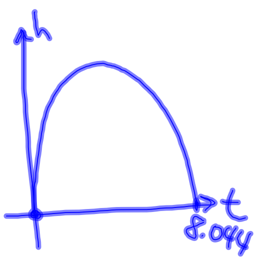
d) When does the ball hit the ground?

ball hits ground when $h=0$

option 1

$0 = -4.9t^2 + 39.2t + 1.75$

→ QF



option 2

$$0 = -4.9(t-4)^2 + 80.15$$

$$\frac{-80.15}{-4.9} = \frac{-4.9(t-4)^2}{-4.9}$$

$$16.357 \doteq (t-4)^2$$

$$t-4 \doteq \pm 4.044$$

$$+4 \quad +4$$

$$t \doteq 4 \pm 4.044$$

$t \doteq 8.044$ or $t \doteq -0.044$

∴ it hits the ground after 8 seconds

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Ex. 3) Supermarket cashiers try to memorize current sale prices while they work. A study showed that, on average, the percent, P, of prices memorized after t hours is given approximately by the formula

$$P = -40t^2 + 120t$$

What is the greatest percent of prices memorized, and how long does it take to memorize them?

optimal value
y-coord of V

x-coord of Vertex

option 1 - vertex form

$P = -40t^2 + 120t + 0$

$P = -40(t^2 - 3t)$

$-\frac{3}{2} = -1.5 \quad (-1.5)^2 = 2.25$

$P = -40(t^2 - 3t + 2.25 - 2.25)$

$P = -40[(t-1.5)^2 - 2.25]$

$P = -40(t-1.5)^2 + 90$

option 2 - factor to find zeroes

set $P = 0$

$0 = -40t^2 + 120t$

$0 = -40t(t-3)$

$t = 0$ or $t - 3 = 0$

$t = 3$

$t_v = \frac{0+3}{2}$

$= 1.5$

for P_v, sub $t_v = 1.5$

$P_v = -40(1.5)^2 + 120(1.5)$

$= -40(2.25) + 180$

$= 90$

∴ the max. memorized is 90% after 1.5 hours

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Assigned Work:

p. 356 #2, 3, 6, 8

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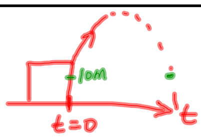
2. $h = 15 + 22t - 5t^2$

$h = -5t^2 + 22t + 15$



(a) set $t=0$ (y-int)

$h = 15$ ∴ height of school is 15m

(b) want to know t when $h=10$

$10 = -5t^2 + 22t + 15$

$0 = -5t^2 + 22t + 5$

$a=-5 \quad b=22 \quad c=5$

$$t = \frac{-(22) \pm \sqrt{22^2 - 4(-5)(5)}}{2(-5)}$$

$$= \frac{-22 \pm \sqrt{484 + 100}}{-10}$$

$$= \frac{-22 \pm \sqrt{584}}{-10}$$

$t = \frac{-22 + \sqrt{584}}{-10} \doteq -0.21$ $\qquad$ $t = \frac{-22 - \sqrt{584}}{-10} \doteq 4.6$

∵ time is positive

∴ rocket at 10m after 4.6s

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2(c) for zeroes, set $h=0$

$0=-5t^2+22t+15$

$a=-5 \quad b=22 \quad c=15 \rightarrow$ QF.

$0=5t^2-22t-15$

S : -22
P : -75
I : $-25, 3$

$0=5t^2-25t+3t-15$

$0=5t(t-5)+3(t-5)$

$0=(t-5)(5t+3)$

$t-5=0$

$t=5$

$5t+3=0$

$5t=-3$

$t=-\frac{3}{5}$

negative time inadmissible

$\therefore$ it hits the ground after 5 s.

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(d) know zeroes at $-\frac{3}{5}$ and 5

$$t_v=\frac{-\frac{3}{5}+5}{2}$$

$$=\frac{-0.6+5}{2}$$

$$=\frac{4.4}{2}$$

$$=2.2$$

sub $t=2.2$ into $h=-5t^2+22t+15$

$h_v=-5(2.2)^2+22(2.2)+15$

$h_v=39.2$

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3. $H=-0.011x^2+0.99x+1.6$

(a) $H=-0.011(x^2-90x)+1.6$ $\qquad \frac{-90}{2}=-45 \qquad (-45)^2=2025$

$H=-0.011(x^2-90x+2025-2025)+1.6$

$H=-0.011[(x-45)^2-2025]+1.6$

$H=-0.011(x-45)^2+22.275+1.6$

$H=-0.011(x-45)^2+23.875$

$\therefore$ the max height is $23.\dot{9}$ m

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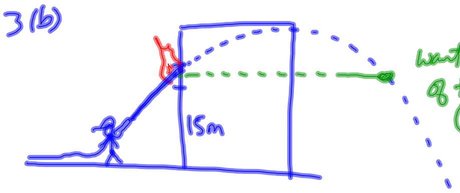
3(b)



want "zeroes" of this line (H=15)

set $H=15$

$15=-0.011(x-45)^2+23.875$

$\frac{-8.875}{-0.011}=\frac{-0.011(x-45)^2}{-0.011}$

$806.81=(x-45)^2$

$\pm 28.4 = x-45$

$x=45\pm 28.4$

want max. distance (x)

$x=45+28.4$

$x=73.4$

$\therefore$ the max. distance the firefighter can stand is 73.4 m from building

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6. $h = -4.9t^2 + 21$

(a) starting height $= 21\text{m}$

halfway is 10.5m

set $h = 10.5$ & solve for t

(b) to reach water, $h = 0$

$0 = -4.9t^2 + 21$, solve for t

(c)

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8. $P = 8x^2 - 112x + 570$

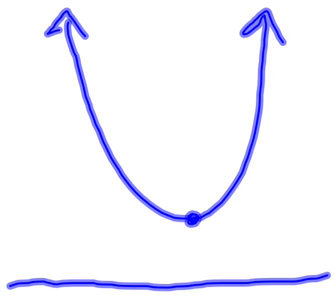
1999 : $x = 0$
2000 : $x = 1$
⋮ ⋮

(a) $1999 \rightarrow x = 0$

$P = 570$

(b) for min, need vertex.

$\rightarrow$ use vertex form



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