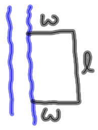
Creating quadratic equations from word problems

Apr. 28/2011

Ex. 1) A rectangular lot is bounded on one side by a river and on the other three sides by 80m of fencing. Determine the dimensions of the largest lot possible.

area



$2w + l = 80$

$A = l \times w$ ②

* want to eliminate l or w from area equation

$2w + l = 80 \rightarrow l = 80 - 2w$ ①

Sub ① into ②

$A = (80 - 2w)(w)$

$A = -2w^2 + 80w$

could find max area by:

① complete square → vertex

② factor → zeroes → midpoint → vertex.

① $A = -2w^2 + 80w$

$A = -2(w^2 - 40w)$

$A = -2(w^2 - 40w + 400 - 400)$

$= -2[(w-20)^2 - 400]$

$= -2(w-20)^2 + 800$

$\frac{-40}{2} = -20$

$(-20)^2 = 400$

$V(20, 800)$

$w = 20,\ l = 80 - 2(20)$

$= 80 - 40$

$= 40.$

∴ dimensions are 20m by 40m

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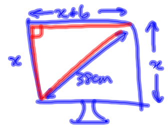
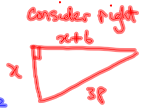
Ex. 2) The size of a television screen or computer monitor is usually stated as the length of the diagonal. A screen has a 38-cm diagonal. The width of the screen is 6 cm more than the height. Find the dimensions of the screen to the nearest tenth.



Let x be the height

consider right



$x^2 + (x+6)^2 = 38^2$

$x^2 + 1(x^2 + 12x + 36) = 38^2$

$2x^2 + 12x + 36 = 1444$

$-1444 \quad -1444$

$2x^2 + 12x - 1408 = 0$

$(x+6)^2$

$= (x+6)(x+6)$

$= x^2 + 6x + 6x + 36$

$= x^2 + 12x + 36$

* to solve a quadratic eqn, find the zeroes

$\frac{2(x^2 + 6x - 704)}{2} = \frac{0}{2}$

$x^2 + 6x - 704 = 0$

$a = 1 \quad b = 6 \quad c = -704$

$x = \frac{-(6) \pm \sqrt{6^2 - 4(1)(-704)}}{2(1)}$

$x = \frac{-6 \pm \sqrt{36 + 2816}}{2}$

$x = \frac{-6 \pm \sqrt{2852}}{2}$

$x \doteq \frac{-6 \pm 53.404}{2}$

$x \doteq 23.702$

∵ dimensions are positive

∴ $x < 0$ is inadmissible

∴ dimensions are height = 23.7 cm

and width = 23.7 + 6

= 29.7 cm

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Ex. 3) The cost of a ticket to a hockey arena which seats 800 people is $3. At this price, every ticket is sold. A survey indicates that for every dollar increase in price, attendance will fall by 100 people. What ticket price results in the greatest revenue? What is the greatest revenue?

$R = (800)(\$3) = 2400$

$R = (700)(4) = 2800$

$R = (600)(5) = 3000$

In general,

$R = (800 - 100x)(3 + x)$

Let x represent the increase in ticket price

to find zeroes, set $R = 0$

$0 = (800 - 100x)(3 + x)$

$800 - 100x = 0$ $\qquad$ $3 + x = 0$

$\frac{800}{100} = \frac{100x}{100}$ $\qquad$ $x = -3$

$x = 8$

$x_v = \frac{8 + (-3)}{2}$

$x = \frac{5}{2}$

$x_v = 2.5$

Sub $x = 2.5$

$R_v = (800 - 100(2.5))(3 + (2.5))$

$= (550)(5.5)$

$= 3025$

$\therefore$ the max revenue of \$3025 occurs when the ticket price is \$5.50.

$y = -100(x - 2.5)^2 + 3025$

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Ex. 4) Determine the number which exceeds its square by the greatest possible amount.

5	25	
2	4	
0.5	0.25	$0.5 - 0.25 = 0.25$
0.1	0.01	$0.1 - 0.01 = 0.09$
x	x^2	$x - x^2 = y$

let x be the number

let y be the difference between the number and its square

$y = x - x^2$

Want y to be a maximum (vertex)

factor

$y = x - x^2$

$y = x(1 - x)$

set $y = 0$

$0 = x(1 - x)$

$x = 0$ or $1 - x = 0$

$x = 1$

$x_v = 0.5$

Sub $x = 0.5$

$y_v = (0.5)(1 - 0.5)$

$= (0.5)(0.5)$

$= 0.25$

$\therefore$ the number is 0.5

cts

$y = -x^2 + x$

$y = (-1)(x^2 - x)$

$= (-1)(x^2 - x + 0.25 - 0.25)$

$= (-1)((x - 0.5)^2 - 0.25)$

$= -(x - 0.5)^2 + 0.25$

$V(0.5, 0.25)$

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Ex. 5) A sporting goods store sells 90 ski jackets in a season for $200 each. Each $10 decrease in price would result in five more jackets being sold.
(a) Find the number of jackets sold and the selling price to give maximum revenues.
(b) What is the lowest price that would produce revenues of at least $15600? How many jackets would be sold at this price?

$$R = (90)(200) = 18000$$
$$R = (95)(190)$$
$$R = (100)(180)$$
$$R = (90+5x)(200-10x)$$

let x be the number of $10 price decreases.

(a) want vertex ① factoring *
② complete square

(b) set $R = 15600$ then solve for x.

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Assigned Work:

p. 158 #14, 15
p. 344 #15, 16
p. 359 #12