Finding Special Lines and Points

Date: _______________

Some tips:

- take your time – if you rush these, you will make mistakes
- draw a neat, labelled diagram – use a ruler, label everything carefully
- label any new points, slopes, or equations – use different colours if possible

A) Slope

Given the equation of a line, put it into the form $y=mx+b$ to find the slope, m.

Given two points, $A(x_1, y_1)$ and $B(x_2, y_2)$, the slope is $m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$.

B) y-intercept

Given the slope, m, of a line, and any point $A(x_1, y_1)$ on the line, substitute the values from the point A into the equation $y=mx+b$ and solve for b.

C) Midpoint

Given two points, $A(x_1, y_1)$ and $B(x_2, y_2)$, the midpoint is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

D) Distance

Given two points, $A(x_1, y_1)$ and $B(x_2, y_2)$, the distance between them is $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.

E) Solving a System of Equations

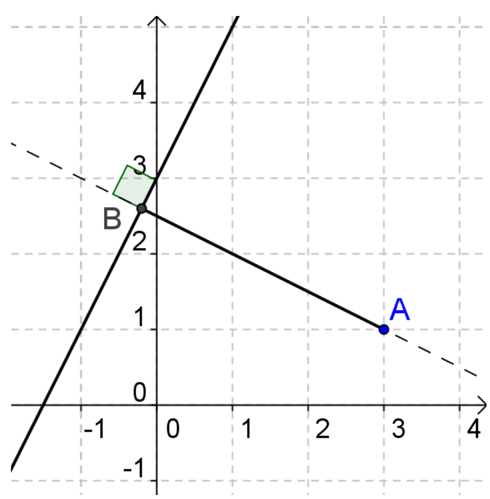
Given two equations in the form $y=mx+b$ or $Ax+By+C=0$, you can solve the system of two equations for the point of intersection, (x, y).

You can solve a system of equations *graphically* by graphing.
You can solve a system of equations *algebraically* by substitution or elimination.

F) Distance From a Point to a Line

The *shortest distance* between a point and line is the *perpendicular distance*.

1. Determine the slope of the perpendicular line using the negative reciprocal of the given line ($m_{AB}=-\frac{1}{m_{given}}$).
2. Sub the known point (A) into the equation you have so far ($y=m_{AB}x+b$) and solve for the y-intercept.
3. Find the point of intersection (B) between the given line and your perpendicular line by solving the system of equations.
4. Determine the distance between your two points (A & B).



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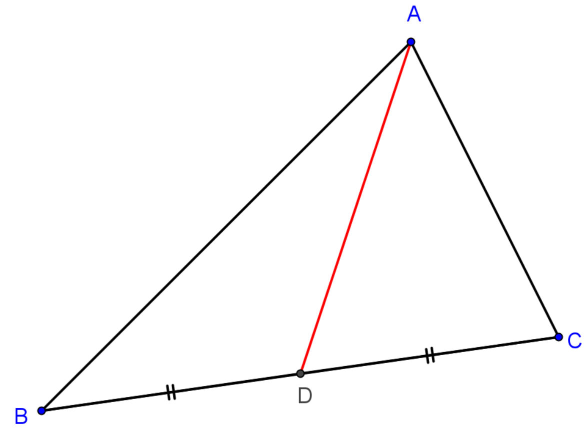
Special Lines

Median – A line that joins a *vertex* of a triangle to the *midpoint* of the opposite side

1. Find the midpoint of the opposite side using two points and give it a label (point D).
2. Find the slope between the vertex and the midpoint (m_{AD}).
3. Write the equation that you know so far ($y=m_{AD}x+b$).
4. Use the slope and the coordinates of either the vertex or midpoint to solve for the y-intercept (sub A or D into $y=m_{AD}x+b$).

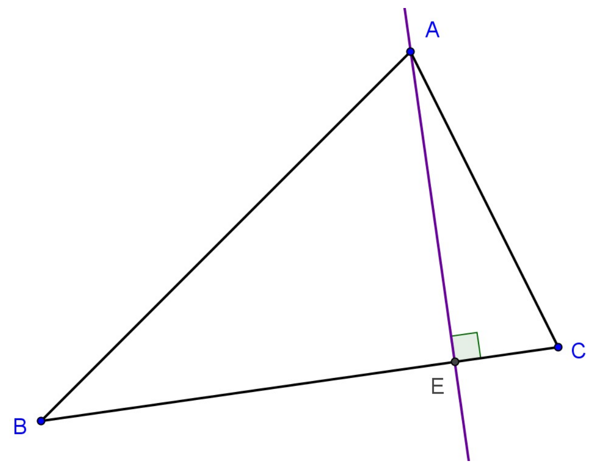
You now have the equation of the median



Altitude – The line segment representing the *height of a polygon*, drawn from a *vertex* to the opposite side so the *line is perpendicular* to the opposite side.

1. Find the slope of the opposite side using two points (m_{BC}).
2. Determine the perpendicular slope by taking the negative reciprocal of your slope from step#1 ($m_{AE}=-\frac{1}{m_{BC}}$). (remember to label these carefully so you don't get them confused)
3. Write the equation that you know so far ($y=m_{AE}x+b$).
4. Sub the coordinates of your vertex (A) into your equation to solve for the y-intercept.

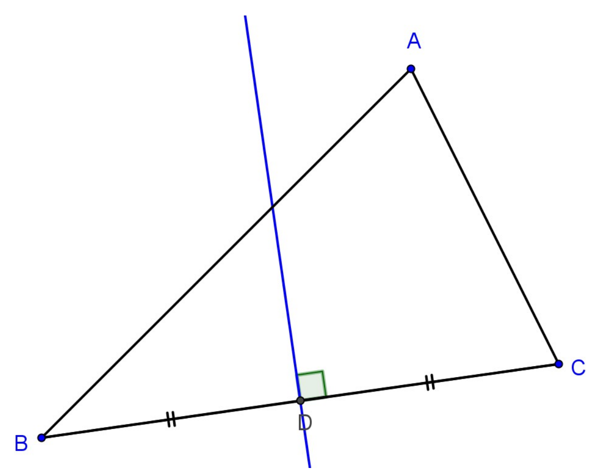
You now have the equation of the altitude



Perpendicular Bisector – A line that is *perpendicular* to a line segment and passes through the *midpoint* of the line segment.

1. Find the midpoint (D) of the line segment using two points (BC).
2. Find the slope of the line segment using the same two points (m_{BC}).
3. Determine the perpendicular slope by taking the negative reciprocal of the slope from step#2 ($m_{\perp}=-\frac{1}{m_{BC}}$). (remember to label these carefully so you don't get them confused)
4. Write the equation that you know so far ($y=m_{\perp}x+b$).
5. Sub the midpoint from step#1 into your equation to solve for the y-intercept.

You now have the equation of the perpendicular bisector



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Centres in Triangles:

Centroid – the point where the **medians** of a triangle meet

1. Each triangle has three medians, and you will **need two of them** to find the centroid. Using the steps for **medians**, you need to:
   a) Find the equation of the any median from any vertex.
   b) Find an equation for a second median from any other vertex.

   You now have two equations in the form $y=mx+b$. Write them out clearly!

2. Solve your system of equations for the point of intersection, (x, y).

   You have just determined the **centroid** of your triangle

Circumcentre – the point where the **perpendicular bisectors** of a triangle meet

1. Each triangle has three perpendicular bisectors, and you will **need two of them** to find the circumcentre. Using the steps for **perpendicular bisectors**, you need to:
   a) Find the equation of the perpendicular bisector from any side.
   b) Find an equation for a second perpendicular bisector from any other side.

   You now have two equations in the form $y=mx+b$. Write them out clearly!

2. Solve your system of equations for the point of intersection, (x, y).

   You have just determined the **circumcentre** of your triangle

Orthocentre – the point where the **altitudes** of a triangle meet

1. Each triangle has three altitudes, and you will **need two of them** to find the orthocentre. Using the steps for **altitudes**, you need to:
   a) Find the equation of the altitude from any vertex.
   b) Find a second equation for the altitude from any other vertex.

   You now have two equations in the form $y=mx+b$. Write them out clearly!

2. Solve your system of equations for the point of intersection, (x, y).

   You have just determined the **orthocentre** of your triangle