

## MHF4U – Unit 2

### Lesson 2 Worksheet: Average vs Instantaneous Rates of Change

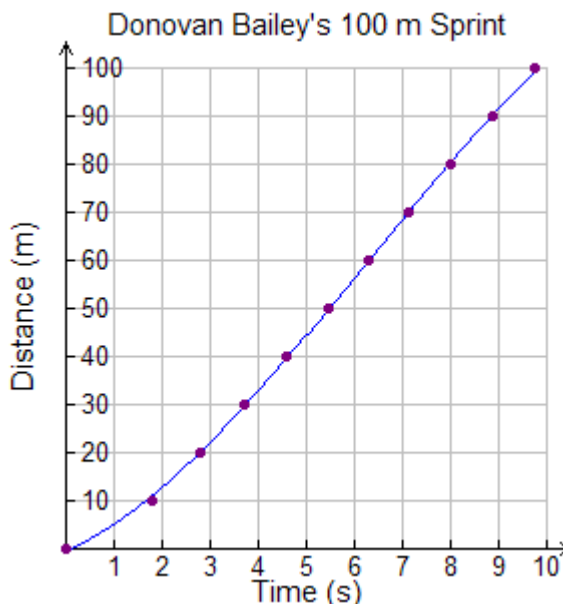
During the 1997 World Championships in Athens, Greece, Maurice Greene and Donovan Bailey ran a 100 m race.

(<http://hypertextbook.com/facts/2000/KatarzynaJanuszkiewicz.shtml>)

#### Part A

The graph and table below show Donovan Bailey's performance during this 100 m race.

| Donovan Bailey's Performance |              |
|------------------------------|--------------|
| Time (s)                     | Distance (m) |
| 0                            | 0            |
| 1.78                         | 10           |
| 2.81                         | 20           |
| 3.72                         | 30           |
| 4.59                         | 40           |
| 5.44                         | 50           |
| 6.29                         | 60           |
| 7.14                         | 70           |
| 8.00                         | 80           |
| 8.87                         | 90           |
| 9.77                         | 100          |



1. a) Calculate Donovan Bailey's average velocity for this 100 m sprint.

$$\text{Average Velocity} = \frac{\text{change in distance}}{\text{change in time}} = \frac{\Delta d}{\Delta t}$$

- b) Draw a line from (0,0) to (9.77,100) on the graph above.

A line passing through at least two different points on a curve is called a **secant**.

- c) Explain the relationship between your answer to a) and the slope of the secant.

2. a) Draw the secants from (0,0) to (5.44,50) and from (5.44,50) to (9.77,100).

- b) Calculate the average velocities represented by the two secants drawn in a).

i)

ii)

- c) Compare Bailey's performance during the first and the second half of the race.

3. Describe the relationship between average velocity and the slope of the corresponding secant.

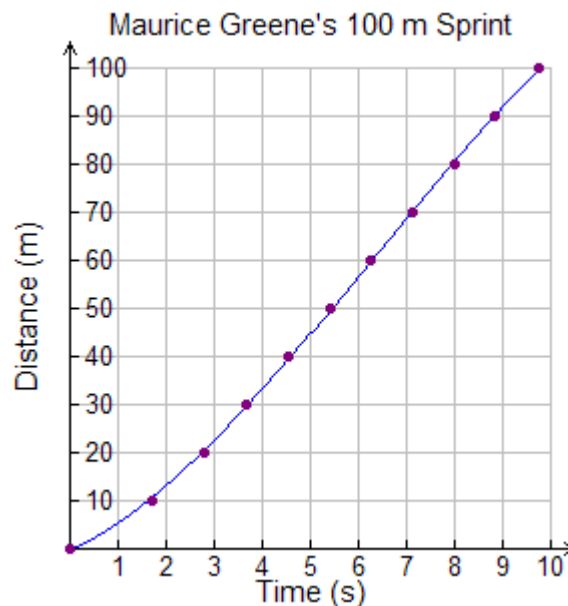
4. Calculate Bailey's average velocity for each 10 m interval of this 100 m race. Record your answers in the table below.

| Interval (m) | Distance Travelled $\Delta d$ (m) | Time Elapsed $\Delta t$ (s) | Average Velocity (m/s) |
|--------------|-----------------------------------|-----------------------------|------------------------|
| 0 to 10      |                                   |                             |                        |
| 10 to 20     |                                   |                             |                        |
| 20 to 30     |                                   |                             |                        |
| 30 to 40     |                                   |                             |                        |
| 40 to 50     |                                   |                             |                        |
| 50 to 60     |                                   |                             |                        |
| 60 to 70     |                                   |                             |                        |
| 70 to 80     |                                   |                             |                        |
| 80 to 90     |                                   |                             |                        |
| 90 to 100    |                                   |                             |                        |

### Part B

The graph and table show Maurice Greene's performance during the same 100 m race.

| Maurice Greene's Performance |              |
|------------------------------|--------------|
| Time (s)                     | Distance (m) |
| 0                            | 0            |
| 1.71                         | 10           |
| 2.75                         | 20           |
| 3.67                         | 30           |
| 4.55                         | 40           |
| 5.42                         | 50           |
| 6.27                         | 60           |
| 7.12                         | 70           |
| 7.98                         | 80           |
| 8.85                         | 90           |
| 9.73                         | 100          |



Calculate Greene's average velocity for each 10 m interval of this 100 m race. Record your answers in the table below.

| Interval (m) | Distance Travelled $\Delta d$ (m) | Time Elapsed $\Delta t$ (s) | Average Velocity (m/s) |
|--------------|-----------------------------------|-----------------------------|------------------------|
| 0 to 10      |                                   |                             |                        |
| 10 to 20     |                                   |                             |                        |
| 20 to 30     |                                   |                             |                        |
| 30 to 40     |                                   |                             |                        |
| 40 to 50     |                                   |                             |                        |
| 50 to 60     |                                   |                             |                        |
| 60 to 70     |                                   |                             |                        |
| 70 to 80     |                                   |                             |                        |
| 80 to 90     |                                   |                             |                        |
| 90 to 100    |                                   |                             |                        |

### Part C

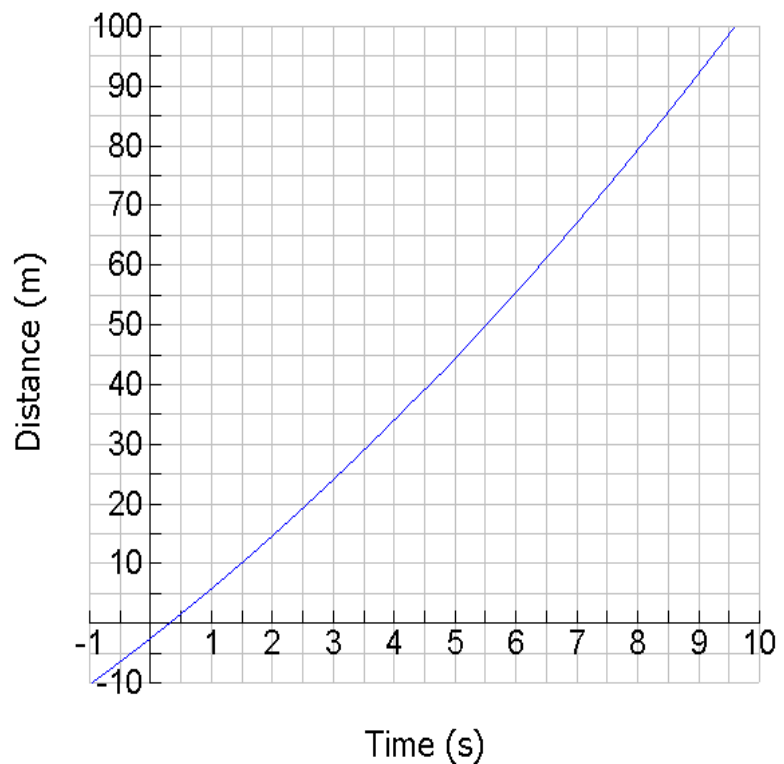
Using your calculations from Parts A and B, describe this 100 m race run by Donovan Bailey and Maurice Greene. Include who was fastest and who was leading at various points during the race.

In 1997, Donovan Bailey ran the 100 m sprint in 9.77 seconds. The table below describes his run. One model that describes this run is a quadratic model with an equation of:

$$d(t) = 0.28t^2 + 8.0t - 2.54.$$

| Time (s) | Distance (m) |
|----------|--------------|
| 0        | 0            |
| 1.78     | 10           |
| 2.81     | 20           |
| 3.72     | 30           |
| 4.59     | 40           |
| 5.44     | 50           |
| 6.29     | 60           |
| 7.14     | 70           |
| 8.00     | 80           |
| 8.87     | 90           |
| 9.77     | 100          |

1. a)



- a) Estimate Donovan Bailey's **instantaneous** velocity at  $t = 6$  s.
- b) Explain why you think your answer to a) is a good approximation.
- c) Plot a point on the curve at 6 seconds. Draw a line that passes through this point but does not pass through the curve again. This line is called a **tangent** to the curve.

2. Use the algebraic model  $d(t) = 0.28t^2 + 8.0t - 2.54$  to approximate the instantaneous velocity of Donovan Bailey at  $t = 6$  s, by completing the charts below.

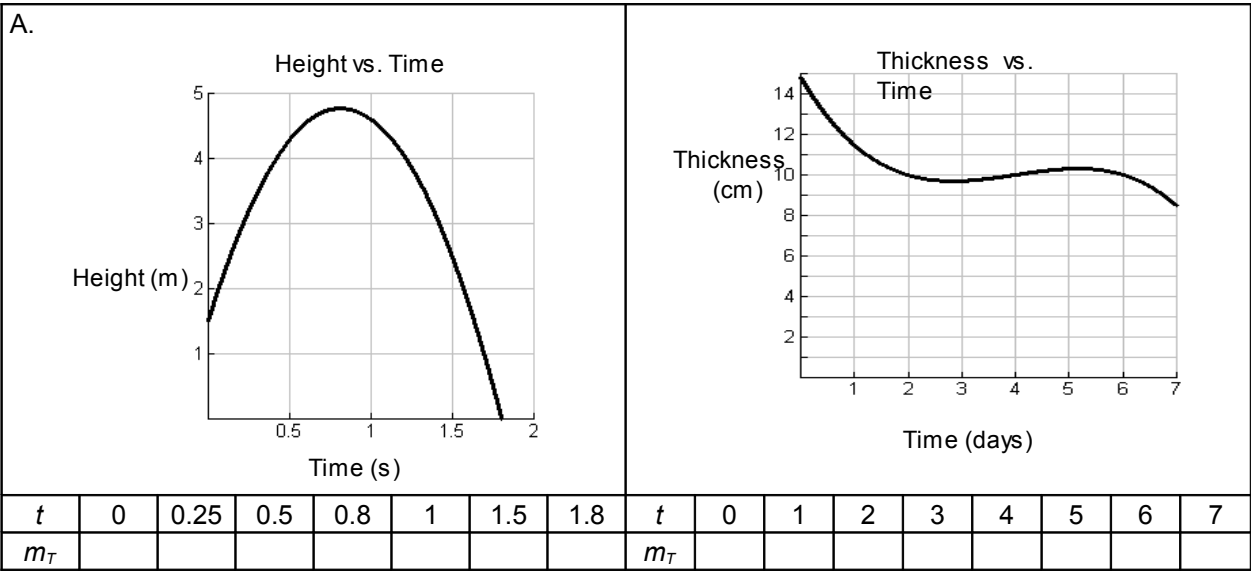
| Interval              | Interval Endpoints | $\Delta d$ | $\Delta t$ | Average Velocity: $\frac{\Delta d}{\Delta t}$ |
|-----------------------|--------------------|------------|------------|-----------------------------------------------|
| $6 \leq t \leq 7$     |                    |            |            |                                               |
| $6 \leq t \leq 6.1$   |                    |            |            |                                               |
| $6 \leq t \leq 6.01$  |                    |            |            |                                               |
| $6 \leq t \leq 6.001$ |                    |            |            |                                               |

| Interval              | Interval Endpoints | $\Delta d$ | $\Delta t$ | Average Velocity: $\frac{\Delta d}{\Delta t}$ |
|-----------------------|--------------------|------------|------------|-----------------------------------------------|
| $5 \leq t \leq 6$     |                    |            |            |                                               |
| $5.9 \leq t \leq 6$   |                    |            |            |                                               |
| $5.99 \leq t \leq 6$  |                    |            |            |                                               |
| $5.999 \leq t \leq 6$ |                    |            |            |                                               |

- i) Use the calculations from the charts to estimate the instantaneous velocity at  $t = 6$  s.
- j) If you could draw the secants that correspond with the average velocities calculated above, how would they compare to the tangent drawn in 1(c)?

MHF 4U Tangent Slopes and Graph Characteristics

1. Using the graphs below, estimate the instantaneous rates of change ( $m_T$ ) for each of the graphs at the given points.



2. State the domain and the range of the two functions.

**Graph A**

Domain:

Range:

**Graph B**

Domain:

Range:

- 3 a) Describe the graphical features (e.g., local maximum/minimum point, interval of increase/decrease),  $m_T$  values (e.g., +, -, 0), and, where appropriate, the trend of the slope of the tangent (e.g., changing from positive to zero to negative) as time increases.

| Interval                 | Graphical feature | $m_T$ values | $m_T$ trend (if appropriate) |
|--------------------------|-------------------|--------------|------------------------------|
| Graph A: Domain 0–0.8    |                   |              |                              |
| Graph A: at 0.8          |                   |              |                              |
| Graph A: Domain 0.8–1.75 |                   |              |                              |
| Graph B: Domain 0–3      |                   |              |                              |
| Graph B: at 3            |                   |              |                              |
| Graph B: Domain 3–5      |                   |              |                              |
| Graph B: at 5            |                   |              |                              |
| Graph B: Domain 5–7      |                   |              |                              |

- b) Describe the context where the slope of the tangent is zero. What does it mean?

- c) What are the similarities and differences between the slopes of the tangents?

4. The slope of the secant line can be a good estimate of the slope of the tangent. Rod thought that using an interval of 1 second to determine the slope of the secant line in graph A is good enough to estimate the slope of the tangent. Do you agree or disagree? Justify your reasoning.
  
  
  
  
  
  
  
  
  
  
5. For Graph A, state the slope of the tangent at 0.5 and 1 second. At which point is the shot put going faster? Explain.
  
  
  
  
  
  
  
  
  
  
6. Draw three or four curves that have a secant slope of 2 and that pass through the same two points. What inference can be made from this?
  
  
  
  
  
  
  
  
  
  
7. Anne says that a tangent crosses a curve in one and only one point. Do you agree or disagree? Use Graph B to justify your position.