Recall: Given $y=a\ f\left[k\left(x-p\right)\right]+q$, the transformations are:

1. vertical reflection in the x-axis if $a<0$
2. vertical scaling by a if $a\neq 1$ (ignoring sign of a)
   vertical stretch by a if $a>1$, vertical compression by $\frac{1}{a}$ if $0<a<1$
3. horizontal reflection in the y-axis if $k<0$
4. horizontal scaling by $\frac{1}{k}$ if $k\neq 1$ (igoring sign of k)
   horizontal stretch by 1 over k if $0<k<1$, horizontal compression by k if $k>1$
5. horizontal translation (shift) by p
6. vertical translation (shift) by q

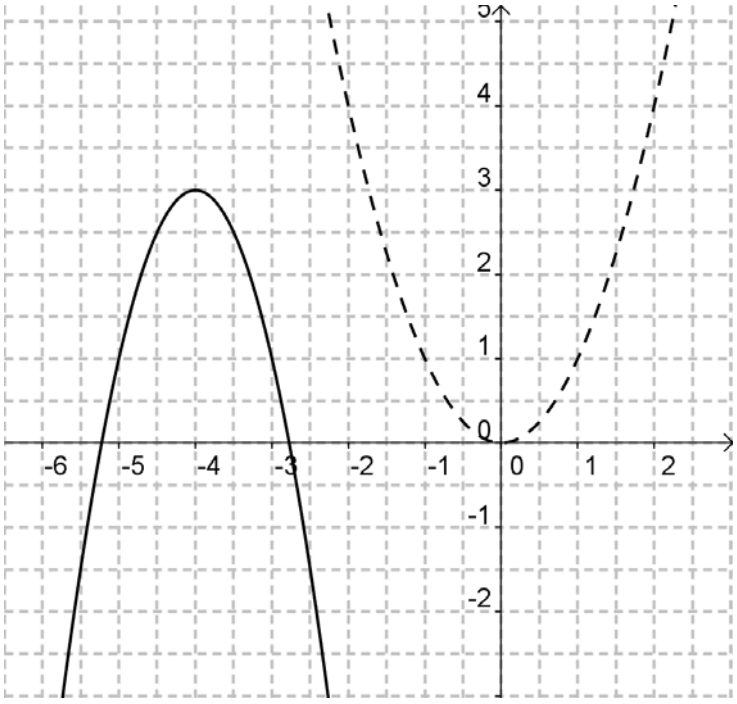
Using these transformations, we can start with any parent function, $y=f\left(x\right)$, and determine the coordinates and the graph of the transformed function, $y=a\ f\left[k\left(x-p\right)\right]+q$.

Each coordinate, $\left(x,y\right)$, will be transformed according to: $\left(x,y\right)\rightarrow\left(\frac{x}{k}+p,a\ y+q\right)$

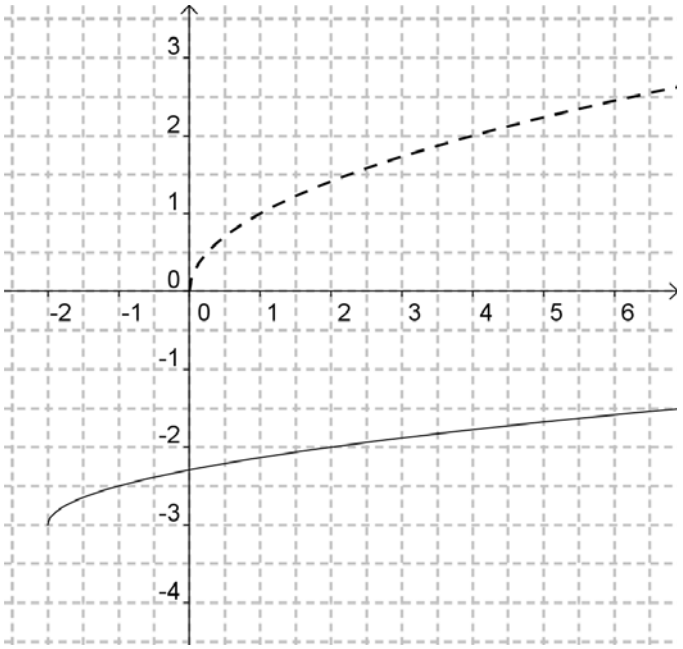
Now let us consider the situation where we are given the graph of the parent function (or relation), and the transformed function (or relation), and we are asked to determine the transformations involved (i.e., finding the values of a, k, p, and q).

Ex.1 The diagram below shows a transformed quadratic function (solid line) as well as the parent function (dotted line). Determine the transformations involved and express the entire transformation using function notation ($y=a\ f\left[k\left(x-p\right)\right]+q$).

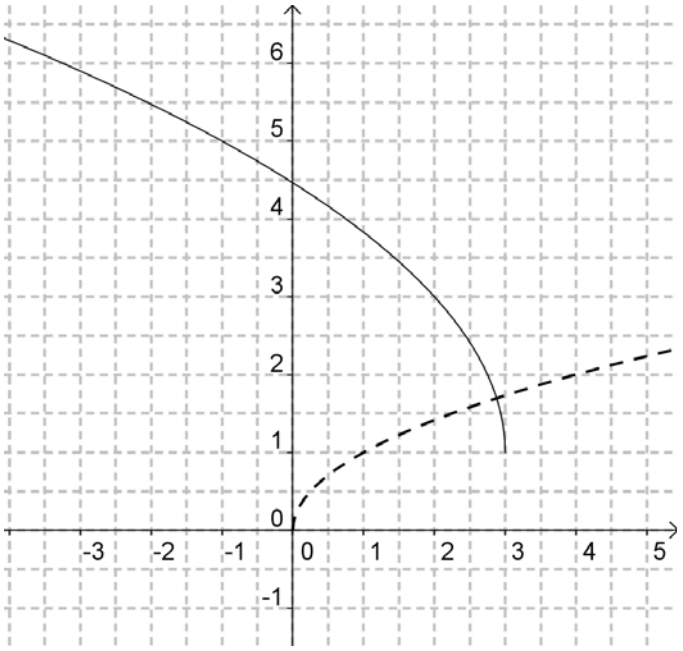
Hint: Consider the vertex form of a quadratic function and ignore any horizontal reflections or scaling.



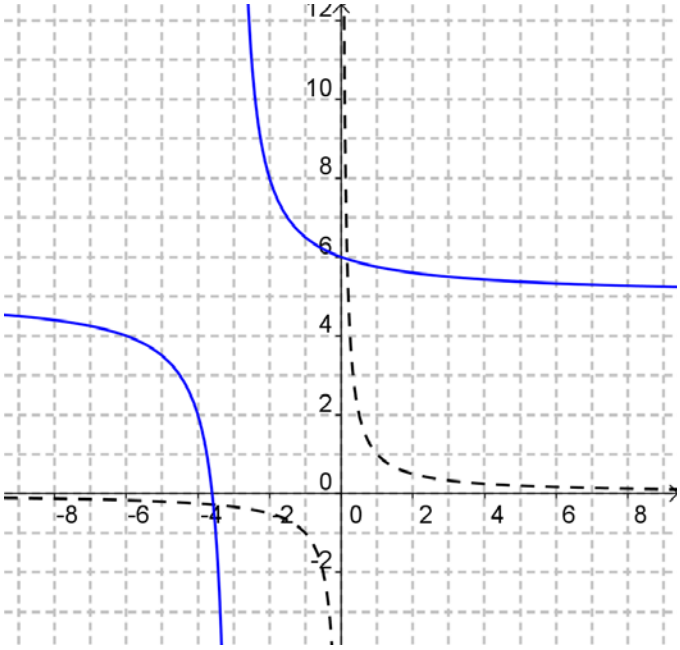
Ex.2 The diagram below shows a transformed radical function as well as the parent function. Determine the transformations involved and express the entire transformation using function notation.
Hint # 1: In a radical function, only vertical scaling is necessary (although horizontal is often convenient)
Hint # 2: The key point, (0, 0), would only be affected by translations (shifts), not reflections or scaling



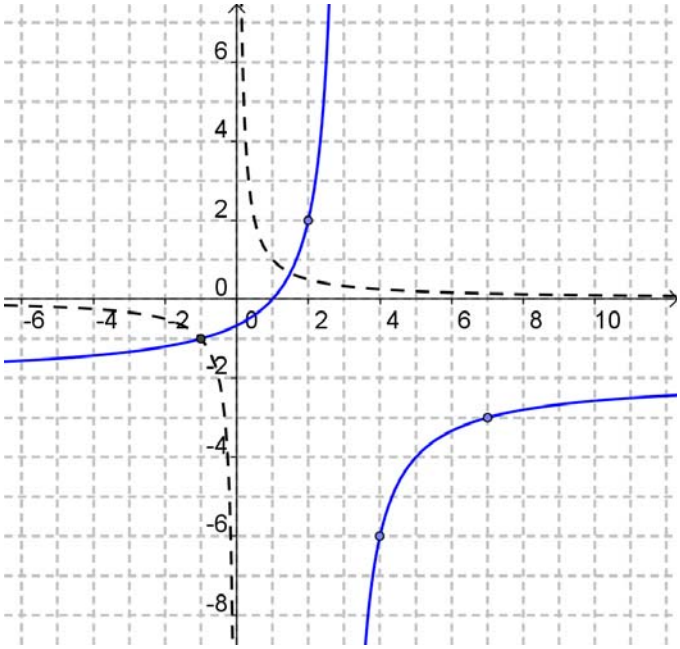
Ex.3 The diagram below shows a transformed radical function as well as the parent function. Determine the transformations involved and express the entire transformation using function notation.
Hint # 1: If a horizontal reflection is present, this **must** be represented in the k-term.
Hint # 2: Any other scaling can be represented as a vertical scaling, as in Ex.2 above



Ex.4 The diagram below shows a transformed reciprocal function as well as the parent function. Determine the transformations involved and express the entire transformation using function notation.
Hint # 1: In a reciprocal function, only vertical scaling is necessary.
Hint # 2: The asymptotes are only affected by translations (shifts), not reflections or scaling



Ex.5 The diagram below shows a transformed reciprocal function as well as the parent function. Determine the transformations involved and express the entire transformation using function notation.
Hint # 1: In a reciprocal function, vertical and horizontal reflections yield the same result.



Ex.6 The diagram below shows a transformed piecewise function as well as the parent function. Determine the transformations involved and express the entire transformation using function notation.

Strategy #1:

a) determine any vertical or horizontal reflections by inspection
b) determine any vertical or horizontal scaling by comparing the rise or run of various line segments
c) once a and k are known, use $(x, y) \rightarrow \left(\frac{x}{k} + p, a\ y + q\right)$ to determine p and q.

Strategy #2:

- Use $(x, y) \rightarrow \left(\frac{x}{k} + p, a\ y + q\right)$ and several points to form and solve systems of equations.