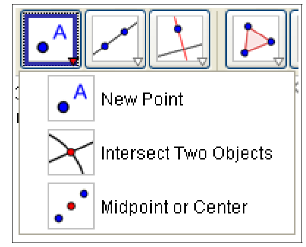
Problem Statement: Triangle STU has vertices at S(-2, -3), T(9, 4) and U(11, -4).

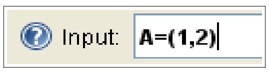
1. Find the equation of the median from S.
2. Find the equation of the altitude from U.
3. Find the equation of the perpendicular bisector of side TU.
4. What kind of triangle is STU? Justify your answer.

1. Plotting Points on the Cartesian (x-y) Plane



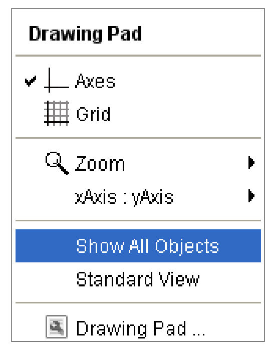
(a) To place points using the mouse, select the **New Point** tool. Any click will produce a new point at that location. The **New Point** tool is located in a pull-down menu with other tools produce a *single point*.

(b) If you are given the coordinates of your point(s), it will probably be simpler to enter them using the keyboard.



At the bottom of the Geogebra screen, there is an **Input bar**, which allows you to perform many actions using input from the keyboard. To create points, you can simply type them into the Input bar using regular coordinate form, (x, y), and then press the `<Enter>` key.

For a bit more control, you can also define the label for the point.



Plot the points S(-2, -3), T(9, 4), and U(11, -4) using the **Input bar** by typing each of the following *exactly as shown*, followed by the `<Enter>` key:

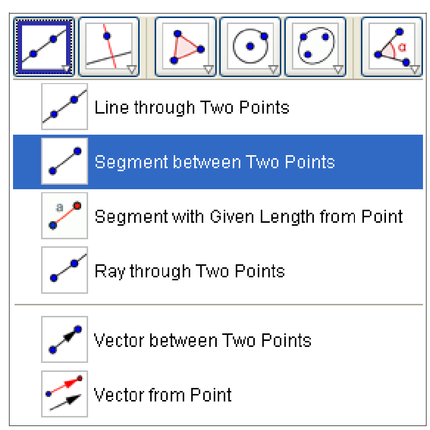
S = (-2, -3)

T = (9, 4)

U = (11, -4)

If you cannot see all three points, you can *right-click* anywhere on the graphic window to produce a special menu. Chose **show all objects** in the menu. The window will shift or change scale so all three points are now visible.

2. Joining Points Using Line Segments



To form geometric figures, we generally use *line segments*, which are straight lines connecting two points. There are a number of other straight constructs that we can produce with *two points*, and all are located within the same drop-down menu.

Find and select the **Segment between Two Points** tool.

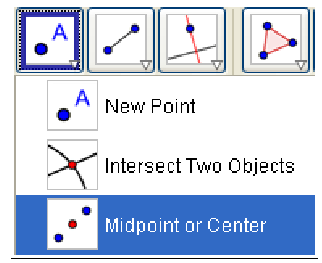
Click on the point S and the point T. A line segment should now be joining the two points.

Join T and U by selecting the point T and the point U.

Join S and U by selecting the point S and the point U.

3. Finding the Midpoint of a Line Segment

We now want to find a *single point*, so we locate the **Midpoint or Center** tool in the same menu as the **New Point** tool. This tool is *context sensitive*, which means it can find the midpoint (a) of a line segment; or (b) between two points.



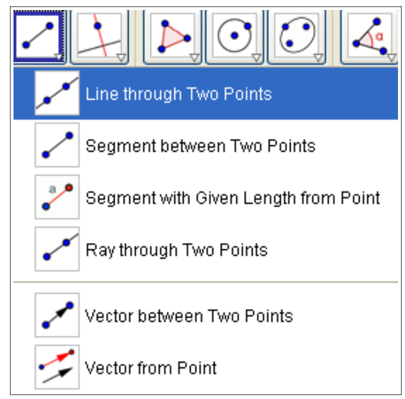
Select the **Midpoint or Center** tool.

Click on the line between Point T and Point U.

4. Creating the Median from S

The median from S is the line segment from the vertex (point) S to the midpoint of the opposite side (the line segment TU).

Find and select the **Segment between Two Points** tool.



Click on the Point S and the midpoint of the line segment TU.

5. Finding the Equation of the Median from S

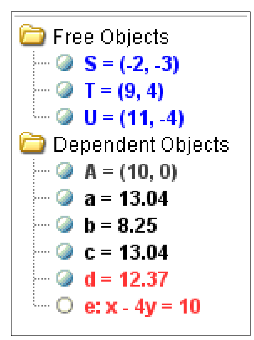
Geogebra can automatically determine the equation of any straight line, but it does not perform this action with *line segments*.

Find and select the **Line through Two Points** tool.

Click on the Point S and the midpoint of the line segment TU.

The equation of the line you just created is displayed in the **Algebra View**, which is generally located at the left side of the program window. Equations of straight lines will be presented in the form $Ax+By=C$.

In this case, the equation of the median from S is $x-4y=10$



By *right-clicking* on the equation, you can also select slope-intercept form, $y=0.25x-2.5$

6. Organization of the Solution

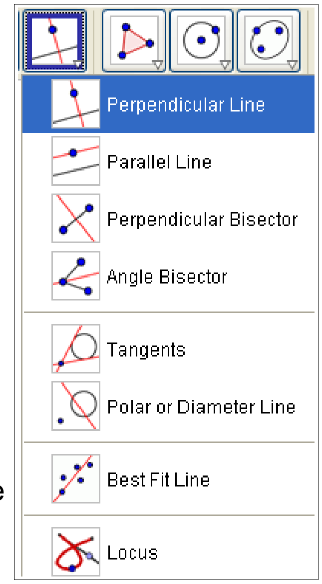
You might want to make some visual changes to improve the organization of your solution. This is particularly important when you are going to have additional parts to your solution (which we are going to do next). You might want to try the following using the *right-click* menu:

Right-click and use **Object Properties** change the colour of the line and line segment to red.

Right-click the line and disable the *Show Object* option. The line will be invisible (but the line segment will still be visible).

You can also make an object visible/invisible by clicking on the circle next to the object in the **Algebra View**. An empty circle means the object is invisible.

7. Creating a Perpendicular Line



The *altitude* from U is a line that passes through the point U, which is also perpendicular to the side opposite to U (which is the line segment ST).

Find and select the **Perpendicular Line** tool.

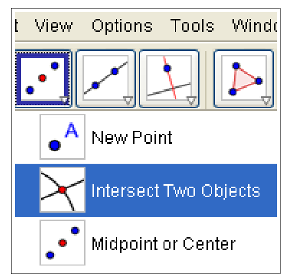
Click on the Point U.

Click on the line segment ST.

Change the colour of the new line to green. Note the equation of this line in the **Algebra View** window to the left.

Technically, the *altitude* should be a *line segment* from U to ST, rather than a line stretching to infinity. To make something more visually appropriate, we need these extra steps:

Find and select the **Intersect Two Objects** tool.



Click on our newly created line (in green) that is *almost* the altitude.

Click on the line segment ST.

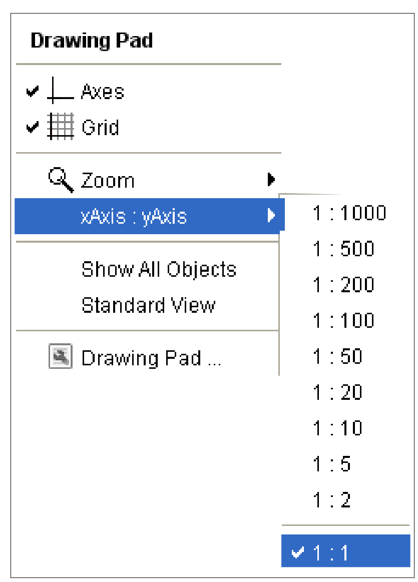
Create a line segment between the newly created point of intersection and the Point U.

Make the infinite line invisible (right-click, Properties).

Change the colour of the altitude to green.

8. Organizing the Solution

Take a close look at the altitude you just created. Does it look like it meets the line segment ST at exactly 90° (a right angle)? When we draw graphs, we often make the scale of the x-axis the same as the scale of the y-axis, but we don't have to do this.



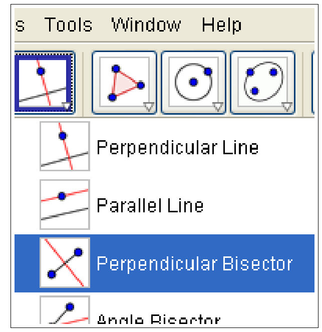
If the x-scale and y-scale are different, visual features such as right angles might not look correct.

Right-click and display the axes and the grid.

Force the x-scale and y-scale to be equal by selecting a **1:1** ratio.

Now might be a good time to experiment with the **Zoom** and the **xAxis:yAxis** ratios. Remember you can always **Show All Objects** to bring your entire drawing back into view.

9. Creating a Perpendicular Bisector



Geogebra has a tool for this important geometric construction. In this exercise, we want the perpendicular bisector of the side TU.

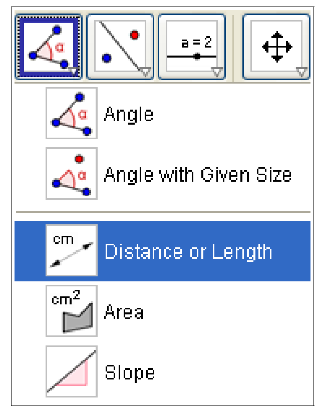
Find and select the **Perpendicular Bisector** tool.

Click on the side TU.

In this case, our perpendicular bisector for TU overlaps with the median from S. This won't always be the case, and it only occurs in this case because of the type of triangle.

10. Determining the Length of a Line Segment

There are three types of triangles, and we generally define them in terms of their side lengths:



(a) Equilateral Triangle – a triangle with all sides having the same length.
(b) Isosceles Triangle – a triangle where two sides have the same length.
(c) Scalene Triangle – a triangle where no sides have the same length.

To identify which type of triangle is STU, we will need a measure of the length of each side.

Find and select the Distance or Length tool.

Click on each of the sides of the triangle: ST, TU, and SU.

Clearly this is an *isosceles triangle.*

You may have noticed that these values were already showing for each line segment in the Algebra View window.

Additional Exercises

1. Given triangle FGH with vertices at F(-7,-1), G(1,6), and H(3,-4):
   (a) Find the coordinates of the circumcentre. Recall that the circumcentre is *equidistant* to each vertex. Can you show this using a circle?
   (b) Find the coordinates of the centroid. You may want to make some of your previous work invisible to keep your diagram uncluttered.
   (c) Find the coordinates of the orthocentre.

2. Show that the mid segments of the quadrilateral with vertices at P(-2, -2), Q(0, 4), R(6, 3), and S(8, -1) form a parallelogram.

3. The Chord Perpendicular Bisector Theorem (CPBT) states that the perpendicular bisector of any chord passes through the centre of the circle. Verify this graphically.

4. The Side-Splitting Theorem (SST) states that the segment formed by joining the midpoints of adjacent sides in a triangle is parallel to and half the length of the third side. Verify this graphically.

5. Use graphical methods to verify the Perpendicular Bisector Theorem (PBT), which states that any point on the bisector of a line segment is equidistant from the endpoints of the line segment.