Annuities

June 8/2011

An annuity is an investment where equal payments (or withdrawls) are made at regular intervals. In an ordinary annuity the payments/withdrawls are made at the end of each interval.

You make payments if you are saving up or paying back a loan and you make withdrawls if you have money saved up and are taking out in equal portions.

The future value of an annuity is the amount of money you will have (saved up) in the future; your deposits plus the interest each of them has earned.

The present value of an annuity represents the amount of money needed to invest **today** in order to provide regular payments/withdrawls over a future period of time.

Jun 7-8:05 PM

Consider an investment of $700 at 4.5%/annnum, compounded annually. How much would this be worth:

(a) after 0 years? 700

(b) after 1 year? $700(1+0.045)$

(c) after 2 years? $700(1+0.045)^2$

(d) after 3 years? $700(1+0.045)^3$

Jun 7-10:53 PM

Consider an investment of $700 at 4.5%/annnum, compounded annually. How much would this be worth:

(a) after 0 years?

(b) after 1 year?

(c) after 2 years?

(d) after 3 years?

What if you had $700 available to invest at the end of <u>each year</u>? How much would you have after 3 years?

Jun 7-10:53 PM

Consider an investment of $700 at 4.5%/annnum, compounded annually. How much would this be worth:

(a) after 0 years?

(b) after 1 year?

(c) after 2 years?

(d) after 3 years?

(e) after n years?

Recall: so

$(1+i)-1$

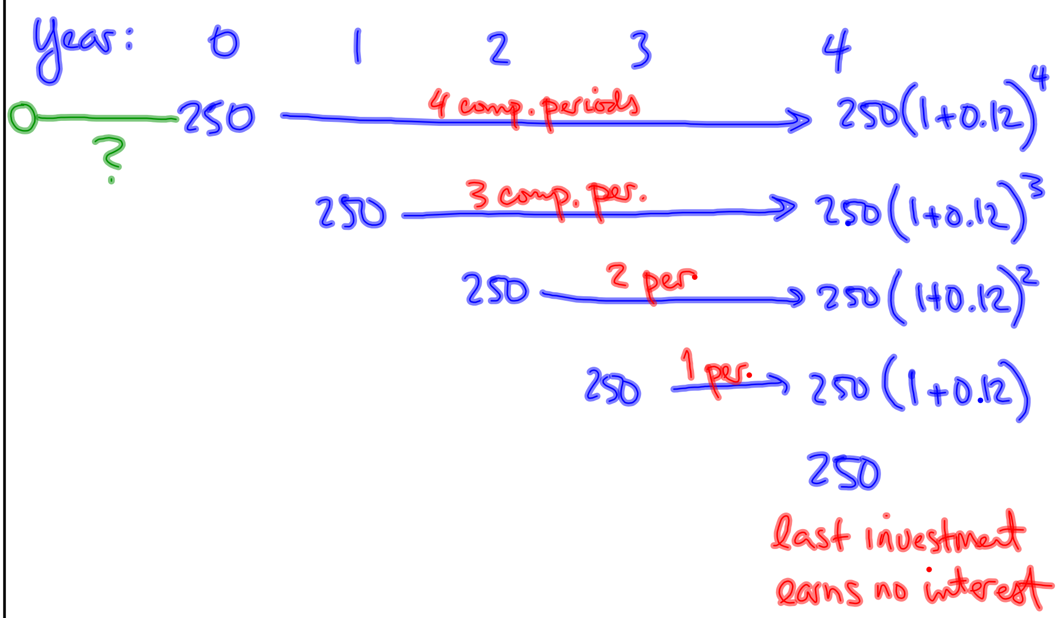
$=1+i-1$

$=i$

Jun 7-10:53 PM

Ex.1 Consider a four year annuity, with 12%/a interest compounded annually, for which you make annual payments of $250.

a) Draw a time line



Jun 7-8:21 PM

Ex.1 Consider a four year annuity, with 12%/a interest compounded annually, for which you make annual payments of $250.

b) Calculate the future value of each payment.

$250(1+0.12)^4 \quad 250(1+0.12)^3$

$250(1+0.12)^2 \quad 250(1+0.12) \quad 250$

c) Determine the future value of the annuity.

$$S_n = \frac{a(r^n - 1)}{r - 1} \Rightarrow FV = \frac{250[(1+0.12)^5 - 1]}{0.12}$$

$$\doteq 1588.21$$

a → 250

r → common ratio $1+0.12$ (100% + 12%)

n → # of terms

Jun 7-8:21 PM

Future value formula:

where R is the regular payment

i is the interest rate (per compounding period)

n is the number of compounding periods (or # of investments)

note: when determining # of compounding periods, recall that for an ordinary annuity, investments occur at the end of each compounding period

Jun 7-8:04 PM

Future value formula:

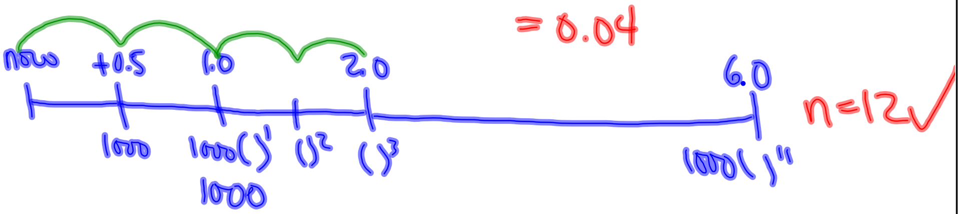
Ex.2 Determine the value of an (ordinary) annuity where $1000 is deposited at the end of each 6 months at 8%/a compounded semi-annually for 6 years.

$n = 12$ ✓

$i = \frac{8\%}{2}$

$= 0.04$

R



$$FV = \frac{1000\left[\left(1+0.04\right)^{12}-1\right]}{0.04}$$

$$FV \doteq 15025.81$$

Jun 7-8:04 PM

Suppose you wanted to have $1000 available 3 years from now. How much should you invest at 3%/a, compounded annually?

$FV = PV(1+i)^n$

$FV = 1000$, $PV = ?$

$$R = \frac{PV \cdot i}{1-(1+i)^{-n}}$$

or

Jun 8-8:16 AM

What about two years?

One year?

Zero years?

What is you wanted to withdraw this amount <u>each year</u> for three years?

now 1year two years 3 years

One of the peculiarities of this type of annuity is that the initial investment is made, and then a withdrawal is made immediately.

Jun 8-8:26 AM

Ex.3 Consider an annuity which provides 9%/a interest compounded annually and provides annual withdrawals of $2000 for four years (after which it is depleted).

a) Draw a time line

now | 1 year | 2 years | 3 years | 4 years

2000

$2000(1+0.09)^{-1}$ ← 2000

$2000(1+0.09)^{-2}$ ← 2000

$2000(1+0.09)^{-3}$ ← 2000

$2000(1+0.09)^{-4}$ ← 2000

total initial investment

Jun 7-8:21 PM

Present value formula:

$$PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$$

n is the number of compounding periods or # of withdrawals (including the first, which occurs immediately)

Ex.4 Determine the present value required for $75 quarterly withdrawals for 10 years at 9.6%/a compounded quarterly.

$n = 10 \times 4$
$= 40$

$i = \frac{9.6\%}{4}$
$= 2.4\%$
$= 0.024$

$R = 75$

$$PV = \frac{75\left[1-(1+0.024)^{-40}\right]}{0.024}$$

$PV \doteq 1914.82$

$\therefore$ $1914.82 needs to be invested now

Jun 7-8:04 PM

Homework:

p.531 #7, 8, 9, 10
p.541 #8, 9, 10

Jun 8-8:49 AM

p.531 #7.

(a) $R = 200$ $i = \frac{0.029}{12}$ $n = 8$

Jun 10-9:10 AM

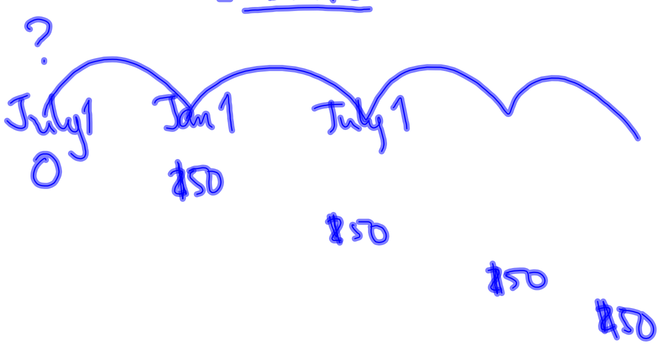
p.531 #9

$R = 150 \quad i = \frac{0.0375}{2} \quad n = 4$

$$FV = \frac{R\left[(1+i)^n - 1\right]}{i}$$

$$= \frac{150\left[\left(1+\frac{0.0375}{2}\right)^4 - 1\right]}{\frac{0.0375}{2}}$$

$$= \underline{617.06}$$



Jun 10-9:12 AM