

Unit 7: Discrete Functions
Arithmetic & Geometric Sequences

May 31/2011

Ex.1 Find the next 3 terms in each sequence:

(a) 3, 7, 11, 15, ... 19, 23, 27, ...
 $+4 +4 +4$

(b) 9, 4, -1, -6, ... -11, -16, -21
 $-5 -5 -5$

(c) 1, 1.25, 1.5, 1.75, ... 2, 2.25, 2.5
 $+0.25 +0.25 +0.25$

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Ex.2 For the sequence 3, 9, 15, 21, ...
1 2 3 4
 $+6 +6 +6$

- (a) find the 100th term.
(b) find a general expression for the nth term.

(a) require 99 steps of +6 to reach t_{100}

$$99(6) = 594$$

$$\text{start at } 3 \rightarrow 3 + 594 = 597$$

$$t_{100} = 597$$

(b) $t_n = 3 + 6(n-1)$

starting value

of steps of a given size

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An arithmetic sequence is a *linear function* where the difference between consecutive terms is a constant (called the common difference, d).

The first term, t_1 , or $f(1)$, is a .

In general, the sequence is:

$$a, a + d, a + 2d, a + 3d, \dots$$

The n^{th} term is:

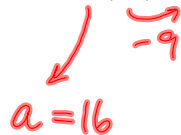
$$t_n = a + (n - 1)d \quad \text{or} \quad f(n) = a + (n - 1)d$$

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Ex.3 How many terms are in the finite sequence

16, 7, -2, -11, ..., -245?

$a = 16$



$$\begin{aligned} t_n &= a + (n-1)d \\ t_n &= 16 + (n-1)(-9) \\ t_n &= 16 - 9(n-1) \end{aligned}$$

Which term is -245?

$$-245 = 16 - 9(n-1)$$

$$\frac{-261}{-9} = \frac{-9(n-1)}{-9}$$

$$29 = n - 1$$

$$n = 30$$

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Ex.4 Find the next three terms in each sequence:

(a) 2, 4, 8, 16, ..., 32, 64, 128, ...

$\xrightarrow{\times 2}$

(b) 1, -2, 4, -8, ..., 16, -32, 64, ...

$\xrightarrow{\times (-2)}$

(c) 27, 9, 3, 1, ..., $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, ...

$\xrightarrow{\div 3 \text{ or } \times \frac{1}{3}}$

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Ex.5 For the sequence 5, 10, 20, 40, ...

- (a) find the 8th term.
(b) write an expression for the nth term.
(c) where would you find 5120 in the sequence?

(a) 5, 10, 20, 40, 80, 160, 320, 640

$$t_8 = 640$$

$$(b) t_n = 5(2)^{n-1}$$

$$(c) \frac{5120}{5} = \frac{5(2)^{n-1}}{5}$$

$$1024 = 2^{n-1}$$

$$2^{10} = 2^{n-1}$$

$$n-1 = 10$$

$$\boxed{n = 11}$$

recall unit on exponential functions

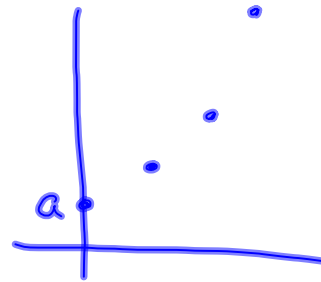
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A geometric sequence occurs when there is a common ratio (r) between consecutive terms.

The first term, t_1 , or $f(1)$, is a .

In general, the sequence is:

$$a, ar, ar^2, ar^3, \dots$$



The n^{th} term is:

$$t_n = ar^{n-1} \quad \text{or} \quad f(n) = ar^{n-1}$$

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Ex.6 Is each sequence geometric? If so, state the common ratio.

(a) $2, -8, 32, -128, \dots$ ✓

$$\text{look at ratios: } \frac{-8}{2} = -4 \quad \frac{32}{-8} = -4 \quad \frac{-128}{32} = -4$$

(b) $x, 2x, 3x, 4x, \dots$ ✗

$$\frac{2x}{x} = 2 \quad \frac{3x}{2x} = \frac{3}{2} \quad \text{ratios are not constant}$$

(c) $x^7, x^{14}, x^{28}, x^{56}, \dots$

$$\frac{x^{14}}{x^7} = x^7 \quad \frac{x^{28}}{x^{14}} = x^{14} \quad \text{✗}$$

(d) $2x^7, 4x^{10}, 8x^{13}, 16x^{16}, \dots$

$$\frac{4x^{10}}{2x^7} = 2x^3 \quad \frac{8x^{13}}{4x^{10}} = 2x^3$$

$$\frac{16x^{16}}{8x^{13}} = 2x^3 \quad \text{✓}$$

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Ex.7 Given $t_5 = 1875$ and $t_7 = 46875$, find t_n (geometric).

In general, $t_n = ar^{n-1}$

$$\begin{aligned} 1875 &= ar^{5-1} & 46875 &= ar^{7-1} \\ 1875 &= ar^4 \text{ ①} & 46875 &= ar^6 \text{ ②} \end{aligned}$$

$$\begin{aligned} \frac{\text{②}}{\text{①}} &\rightarrow \frac{46875}{1875} = \frac{ar^6}{ar^4} \\ &25 = r^2 \\ &r = \pm 5 \end{aligned}$$

why ± 5 ? we don't know if t_6 changes sign.

$$\begin{array}{ll} \text{for } r=5 & \text{for } r=-5 \\ t_n = a(5)^{n-1} & t_n = a(-5)^{n-1} \\ 1875 = a(5)^4 & 1875 = a(-5)^4 \\ 1875 = a(625) & 1875 = a(625) \\ a=3 & a=3 \end{array}$$

$$\therefore t_n = 3(5)^{n-1} \text{ or } t_n = 3(-5)^{n-1}$$

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Assigned Work:

p.441 # [1-4][basics], 5-8(adg), 9, 12, 15, 18
p.452 # [1-4][basics], 5-7(adf), 9, 13, 18

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p. 442 # 6(d) $-7 - (-10) = 3$

$-10, -7, -4, \dots$
+3 +3

t_{11}, t_{22}

$$t_n = a + (n-1)d$$

$$* t_n = -10 + (n-1)(3)$$

$$\begin{aligned} t_{11} &= -10 + (11-1)(3) \\ &= -10 + 30 \\ &= 20 \end{aligned}$$

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8(a) $t_5 = 16$ $t_8 = 25$

$$t_n = a + (n-1)d$$

$$16 = a + (5-1)d \quad 25 = a + (8-1)d$$

$$16 = a + 4d \quad \textcircled{1} \quad 25 = a + 7d \quad \textcircled{2}$$

$$\textcircled{2} - \textcircled{1}: 25 = a + 7d$$

$$\frac{-9}{-3} = \frac{-3d}{-3}$$

$$\boxed{d=3}$$

sub $d=3$ into $\textcircled{1}$

$$16 = a + 4(3)$$

$$16 = a + 12$$

$$\boxed{a=4}$$

$$t_n = 4 + (n-1)(3) \checkmark$$

$$t_n = 4 + 3(n-1) \checkmark$$

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