Applications of Sequences:
Simple & Compound Interest

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Simple interest is only paid on the principal (initial investment) and can be modelled as an arithmetic sequence.

Many GICs (Guaranteed Investment Credits) calculate interest in this way.

Ex.1 $2000 is invested in a 5-year GIC that pays 2.1% per year, calculated annually. What is the value of the investment when it matures?

2000

$2000 + 2000(0.021) = 2042$

$2000 + 2 \times 2000(0.021) = 2084$

$\vdots$

$\underbrace{2000}_{a} + \underbrace{5}_{\text{\# of years}} \times \underbrace{2000(0.021)}_{d} = 2210$

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In general,

$A = P + Prt$ where A is the final amount
P is the initial investment
(or principal)
r is the rate of interest
t is the term (time)

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Compound interest is earned on both the principal as well as any interest earned as the investment grows.

It is now important to consider the compounding period, which is how frequently interest is calculated, as well as the rate of interest (usually quoted per annum, or year).

Savings accounts will generally pay this type of interest.

Ex.2 $5000 is invested for 10 years at 1.5%, compounded annually.

1yr: $5000 + 5000(0.015) = 5000(1+0.015)$

2yr: $\underbrace{\left(5000(1+0.015)\right)}_{\text{last year's total}}\underbrace{(1+0.015)}_{\text{100\% + new interest}}$

$= 5000(1+0.015)^2$

3yr: $5000(1+0.015)^3$

⋮

10yr: $5000(1+0.015)^{10} = 5802.70$

In general,

$$A = P(1+r)^n$$

100% of principal

where A is the final amount
P is the principal invested
r is the rate of interest **per compounding period**
n is the number of compounding periods

Ex.3 Compare the following investment options:
(a) 9 years at 6% per annum, compounded semi-annually
(b) 9 years at 5.95% per annum, compounded monthly

$A = P(1+r)^n$

(a) 9 years, semi-annually → 2× per year
$n = 18$ compounding periods
6% per year → 3% per half-year
$r = 0.03$
choose $P = 1000$

$A = 1000(1+0.03)^{18}$
$A = 1702.43$

(b) $n = \underset{\text{years}}{9} \times \underset{\text{monthly}}{12}$
$= 108$

$r = \frac{0.0595}{12}$
$\doteq 0.004958$

$A = 1000\left(1+\frac{0.0595}{12}\right)^{108}$
$A \doteq 1706.04$

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When people invest, they are generally interested in the <u>present value</u> and <u>future value</u> of the investment.

Knowing one allows the other to be calculated:

$$FV = PV(1+r)^n \qquad \text{or} \qquad PV = \frac{FV}{(1+r)^n}$$

Ex.4 To buy a house 3 years from now, a down payment of \$70000 will be required. How much needs to be invested now, at 5.25%/a, compounded monthly, to meet this goal?

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$n = 3 \times 12$
$= 36$

$r = \frac{0.0525}{12}$

$FV = 70000$

$PV = \frac{FV}{(1+r)^n}$

$= \frac{70000}{\left(1+\frac{0.0525}{12}\right)^{36}}$

$\doteq 59818.83$

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Assigned Work:

p.508 # [2 - 5], 7, 13
p.523 # [2], 6, 7, 10, 15

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