

Mortgages

June 10/2011

A mortgage is a loan you take out which you then pay back with equal payments at regular intervals.

Thus a mortgage is an annuity!

A down payment is a one time payment you make at the time that you borrow the loan so that the amount you borrow is smaller.

An annual interest rate is often referred to as a nominal rate.

In Canada, a standard mortgage compounds its interest semi-annually and the payments are made monthly.

This discrepancy, between compounding period and payment frequency, means you must calculate the equivalent interest rate (the rate for the time period of the payments).

Jun 7-8:05 PM

Consider a 12%/a interest compounded semi-annually, for monthly payments of \$250. How much money will you have after 4 years?

In order to determine the future value of this annuity you would first have to determine the equivalent interest rate.

Convert $(1+i)^n$ for the 12%/a interest, compounded semi-annually, to $(1+i)^n$ if the rate was compounded monthly.

$$\text{semi-annually: } 1 + \frac{0.12}{2} = 1.06$$

$$\rightarrow \text{equivalent annual rate: } (1.06)^2 = 1.1236$$

\therefore 12%/a semi-annual is the same as 12.36% annually.

$$\rightarrow \text{equivalent monthly rate: } i = \frac{0.1236}{12}$$

$$\begin{array}{l} \text{semi-annual} \left\{ \begin{array}{l} 1000(1.06) = 1060 \quad \text{after 6 mo.} \\ 1060(1.06) = 1123.6 \quad \text{after 12 mo.} \end{array} \right. \end{array}$$

$$\text{annual} \left\{ 1000(1.1236) = 1123.6 \quad \text{after 1 year} \right.$$

$$\text{monthly} \left\{ 1000 \left(1 + \frac{0.1236}{12} \right) \right.$$

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Notes on equivalent interest

In general, equivalent interest can be calculated as follows:

1. Convert given interest to an equivalent annual rate:

e.g., 5%/annum, compound semi-annually

$$\left(1 + \frac{0.05}{2}\right)^2 = 1.1025 = 1 + 0.1025$$

so the equivalent annual rate is 10.25%

2. Convert the equivalent annual rate to the required compounding period:

e.g., 5%/a, semi-annual, to monthly

$$(1+i)^{12} = 1.1025$$

$$(1+i) = \sqrt[12]{1.1025} = (1.1025)^{\frac{1}{12}}$$

$$1+i = 1.0081648$$

$$i = 0.0081648$$

This method did not match results obtained using various mortgage calculators. The method discussed in class did match those calculators. Once this discrepancy is reconciled, we will discuss in class.

Jun 11-5:13 PM

A mortgage problem is similar to a present value annuity problem. For example,

PV Problem: How much should I invest now (PV) to provide regular payments (R) over some time period, assuming I can make some interest on my investment (i).

Mortgage Problem: The bank has invested some amount for me (by buying me a house) now (PV), but they expect me to pay back this loan with regular payments (R) over some time period (with interest, i).

$$PV = \frac{R \left[1 - (1+i)^{-n} \right]}{i} \quad \text{or} \quad R = \frac{PV \cdot i}{1 - (1+i)^{-n}}$$

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Ex.1 A house is purchased for \$425 000 with a \$50 000 down payment. A 5%/a mortgage, compounded semi-annually, is taken out for 25 years.

a) Determine the monthly ^{pay back mortgage} interest rate. $(1+i)^n$

$$\text{Semi: } i = \frac{0.05}{2}$$

$$\rightarrow \text{annual: } \left(1 + \frac{0.05}{2}\right)^2 = 1.050625$$

$$\rightarrow \text{monthly: } \left(1 + \frac{0.050625}{12}\right) \quad i = \frac{0.050625}{12}$$

b) Calculate the amount that needs to be borrowed for the mortgage.

$$425000 - 50000 = 375000$$

c) Determine the monthly payments that are to be made.

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Ex.1 (continued)

A house is purchased for \$425 000 with a \$50 000 down payment. A 5%/a mortgage, compounded semi-annually, is taken out for 25 years.

c) Determine the monthly payments that are to be made.

$$R = \frac{PV(i)}{1 - (1+i)^{-n}} \quad i = \frac{0.050625}{12}$$

$$R = \frac{375000 \left(\frac{0.050625}{12}\right)}{1 - \left(1 + \frac{0.050625}{12}\right)^{-300}} \quad PV = 375000$$

$$n = 25 \times 12 = 300$$

$$R = 2205.89$$

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Ex.1 (continued)

A house is purchased for \$425 000 with a \$50 000 down payment. A 5%/a mortgage, compounded semi-annually, is taken out for 25 years.

d) What is the total cost of the mortgage, as compared to the amount borrowed from the bank?

$$\begin{aligned} \text{total cost} &= (2205.89)(300) \\ &= 661767 \end{aligned}$$

e) Determine the total interest paid on the mortgage.

$$\begin{aligned} \text{diff} &= 661767 - 375000 \\ &= 286767 \end{aligned}$$

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Amortization tables are data organized in a way that lets you view the various values of a mortgage over its lifetime.

For each payment, it will generally show you

- (a) how much goes to the principal
- (b) how much goes to interest
- (c) the remaining balance on the principal

<http://www.bretwhissel.net/cgi-bin/amortize>



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Assigned Work:

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Fatima takes out a mortgage of \$200 000 amortized over 25 years. The bank offers a 5.25% interest rate compounded semi-annually for a 10-year term.

- a) Calculate the equivalent interest rate.
- b) Determine the monthly payment.
- c) Determine the total amount paid in 10 years.
- d) Determine the total interest paid in 10 years.

Jun 8-8:02 AM