Recursion Formulae & Recursive Sequences

June 1/2011

Determine the pattern in the Fibonacci sequence:

1, 1, 2, 3, 5, 8, ... 13, 21, 34, ..

$t_5 = t_4 + t_3$

$t_{100} = t_{99} + t_{98}$

$t_n = t_{n-1} + t_{n-2}$ $n \varepsilon \mathbb{N}$, $n \geq 3$

$\underset{100}{n}$ $\underset{99}{n-1}$ $\underset{98}{n-2}$

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A sequence is recursive if a new term is found using a previous term (or terms).

Ex 1) Find the first ~~four~~ three terms in each of the following sequences

a) $t_n = t_{n-1} - 2$, $t_1 = 3$

$t_1 = 3$ $t_2 = t_1 - 2$ $t_3 = t_2 - 2$

$= 3 - 2$ $= 1 - 2$

$= 1$ $= -1$

b) $f(n) = f(n - 1) + 1.5$, $f(1) = -0.5$

$f(2) = f(1) + 1.5$ $f(3) = f(2) + 1.5$ $f(4) = f(3) + 1.5$

$= 1$ $= 2.5$ $= 4$

$f(1000) = f(999) + 1.5$

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Ex 2) Determine a recursion formula for each of the following sequences, then write an explicit formula if possible (an explicit formula does not rely on recursion).

a) -3, 6, -12, 24, ...

$\times -2 \quad \times -2 \quad \times -2$

$+9 \quad \cancel{+9}$

recursive

$t_n = -2t_{n-1}$

$t_1 = -3$

geometric

$t_n = ar^{n-1}$

$t_n = -3(-2)^{n-1}$

b) f(1) = 2, f(2) = 6, f(3) = 10, f(4) = 14, ...

2, 6, 10, 14

$+4 \quad +4$

$\times 3 \quad \times$

$t_n = t_{n-1} + 4$

$f(n) = f(n-1) + 4$

$f(1) = 2$

arithmetic

$t_n = a + (n-1)d$

$t_n = 2 + (n-1)(4)$

$f(n) = 2 + (n-1)(4)$

c) 3, 5, 8, 12, ...

$+2 \quad +3 \quad +4$

$t_2 = t_1 + 2$

$t_3 = t_2 + 3$

$t_4 = t_3 + 4$

$t_n = t_{n-1} + n, \quad t_1 = 3$

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$\checkmark \; t_5 = t_{6-1}$

$t_{100} = t_{101-1}$

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Pascal's Triangle & The Binomial Theorem

Consider:

$(x+y)^0 = 1$

$(x+y)^1 = x+y$

$(x+y)^2 = x^2+2xy+y^2$

$$
\begin{aligned}
(x+y)^3 &= (x+y)(x+y)(x+y) \\
&= (x^2+2xy+y^2)(x+y) \\
&= x^3+x^2y+2x^2y+2xy^2+xy^2+y^3 \\
&= x^3+3x^2y+3xy^2+y^3
\end{aligned}
$$

$(x+y)^7 = ?$

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Pascal's Triangle & The Binomial Theorem

The coefficients from the *binomial expansions* can be organized as a triangle, where each number in the triangle is the sum of the two numbers above it.

$(x+y)^0 = 1$

$(x+y)^1 = x+y$

$(x+y)^2 = x^2+2xy+y^2$

$(x+y)^3 = x^3+3x^2y+3xy^2+y^3$

1	$n=0$
1 1	$n=1$
1 2 1	$n=2$
1 3 3 1	$n=3$
1 4 6 4 1	$n=4$
1 5 10 10 5 1	5
1 6 15 20 15 6 1	6
1 7 21 35 35 21 7 1	7

$(x+y)^7 = x^7+7x^6y+21x^5y^2+35x^4y^3+35x^3y^4+21x^2y^5+7xy^6+y^7$

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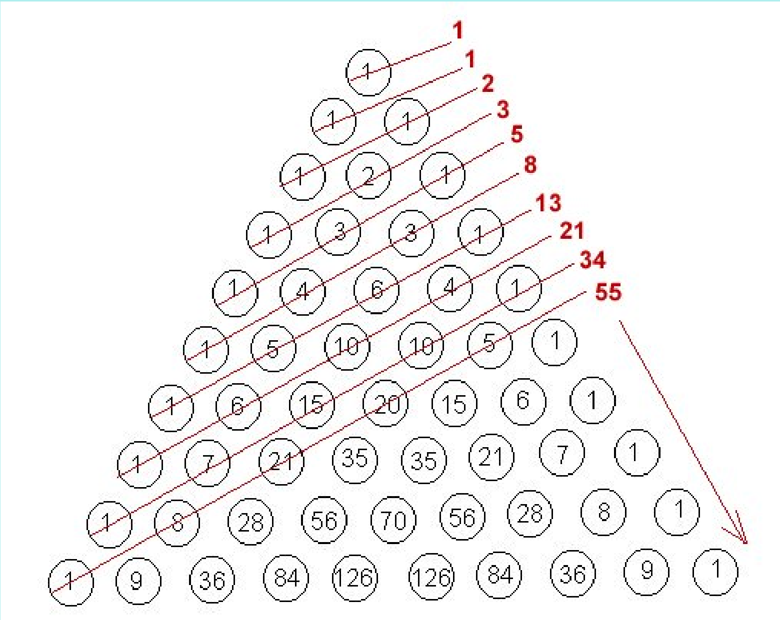
Fibonacci Numbers from Pascal's Triangle

The Fibonacci numbers can be obtained by adding the *shallow diagonals* in Pascal's triangle.



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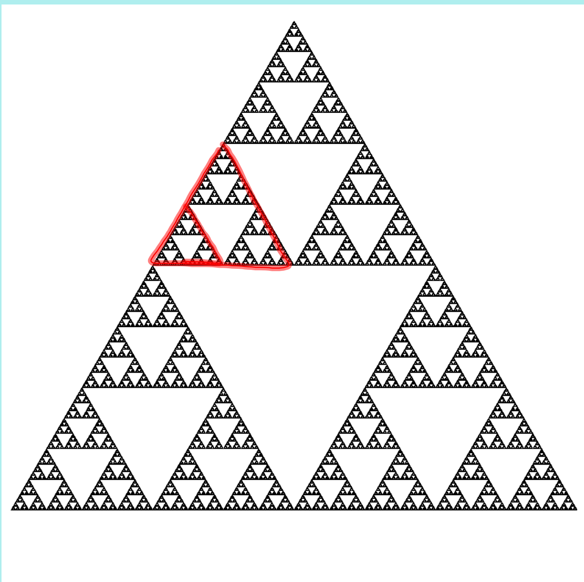
Recursion can also be applied to geometric figures to produce **fractals**, patterns that repeat themselves at any scale (zooming in or out).

One of the simplest and most famous is the Serpenski Triangle:



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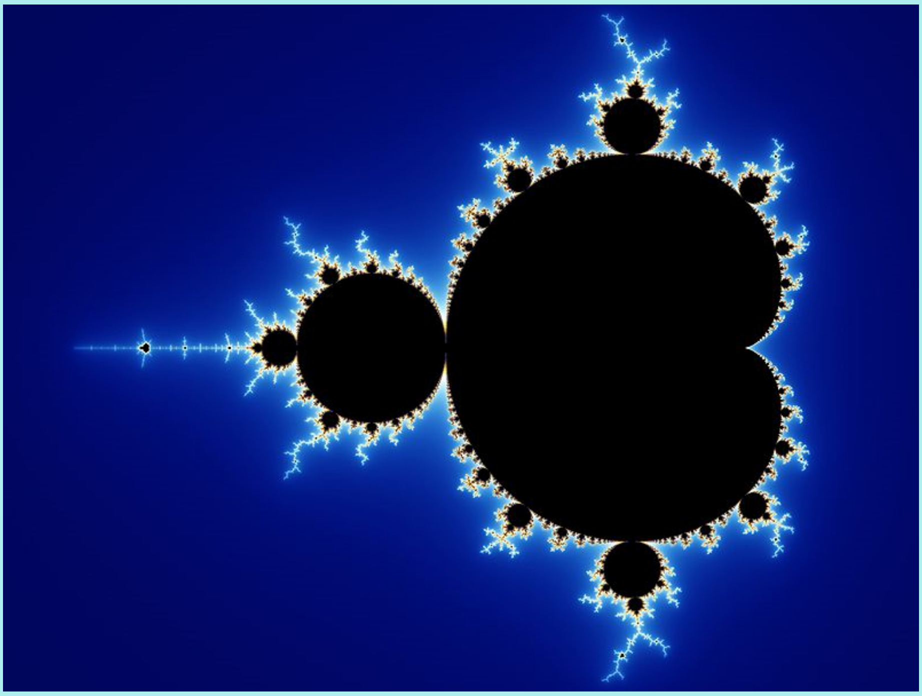
The Mandelbrot Set: A recursively defined set of points



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Assigned Work: P. 461 #[1-5] basics, 8, 9, 10, 12

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8. $t_1 = 5$ $t_n = t_{n-1} + n - 4$

5, 3, 2

-2 -1

$r_1 = \frac{3}{5}$ $r_2 = \frac{2}{3}$

$t_2 = t_1 + 2 - 4$
$= 5 + 2 - 4$
$= 3$

$t_3 = t_2 + 3 - 4$
$= 3 - 1$
$= 2$

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10. $t_1 = 20$ $t_n = t_{n-1} + 2$

$t_2 = t_1 + 2$
$= 20 + 2$
$= 22$

(a) 20, 22, 24, 26, 28, 30, 32, 34

(b) $t_n = a + (n-1)d$
$t_n = 20 + (n-1)(2)$ ✓
$= 20 + 2(n-1)$ ✓
$= 20 + 2n - 2$
$= 18 + 2n$ ok

Jun 2-9:16 AM

p. 442 # 7(a)

10, 15, 20, ..., 250

$t_n = a + (n-1)d$

$t_n = 10 + (n-1)(5)$

$250 = 10 + (n-1)(5)$

$\frac{240}{5} = \frac{(n-1)(5)}{5}$

$48 = n-1$

$n = 49$

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