Add $1 \to 100$

$1, 2, 3, 4, \ldots, 98, 99, 100$

$$\begin{array}{l} 1+2+3+4+\ldots+98+99+100 \\ 100+99+98+97+\ldots+3+2+1 \\ \hline 101+101+101+\ldots+101+101+101 \end{array}$$

$100 \times 101 = 10100$

$\frac{10100}{2} = 5050$

Jun 2-9:21 AM

$$\begin{array}{l} 1+2+3+\ldots+n-2+n-1+n \\ n+n-1+n-2+\ldots+3+2+1 \\ \hline n+1 \quad n+1 \quad n+1 \quad \ldots \quad n+1 \end{array}$$

$\frac{n(n+1)}{2}$

$1 \to 2036$

Jun 2-9:27 AM

Arithmetic and Geometric Series

June 2/2011

Definitions:

1) A <u>series</u> is the **sum** of the terms in a sequence. The sum is denoted S.

e.g.: sequence: $t_1, t_2, t_3, ..., t_n$

series: $t_1 + t_2 + t_3 + ... + t_n = S_n$

$n \varepsilon \mathbb{N}$

note: $S_1 = t_1$

$S_2 = t_1 + t_2$

$S_3 = t_1 + t_2 + t_3$,

etc.

To get from one sum to the next you add the next term, thus

$S_n = S_{n-1} + t_n$ or $S_n - S_{n-1} = t_n$

May 27-2:43 PM

Definitions:

2) An <u>arithmetic series</u> is the sum of the terms in an arithmetic sequence.

The sum can be found using one of the following formulae:

$$s_n = \frac{n}{2}\left[2a + (n-1)d\right] \quad \text{or} \quad s_n = \frac{n}{2}\left[t_1 + t_n\right]$$

Recall: $t_n = a + (n - 1)d$

May 27-2:52 PM

Ex: 1. Determine S_{30} for the arithmetic series

$5 + 9 + 13 + 17 + \ldots + (4n+1)$

① $S_n = \frac{n}{2}[2a+(n-1)d]$

$n=30 \quad a=5 \quad d=+4$

$S_{30} = \frac{30}{2}[2(5)+(29)(4)]$

$= 15[10+116]$

$= 15[126]$

$= 1890$

② $S_n = \frac{n}{2}[t_1+t_n]$

$t_n = 4n+1$ (verify this using known terms)

$t_{30} = 4(30)+1$

$= 121$

$S_{30} = \frac{30}{2}[t_1+t_{30}]$

$= 15[5+121]$

$= 15[126]$

$= 1890$

May 27-2:58 PM

Ex 2) Find the sum of the first 10 terms of the arithmetic series 21 + 16 + 11 + ...

$S_n = \frac{n}{2}[2a+(n-1)d]$

$a = 21$

$d = -5$

$n = 10$

$S_{10} = \frac{10}{2}[2(21)+(9)(-5)]$

$= 5[42-45]$

$= 5[-3]$

$= -15$

May 27-3:02 PM

$$(a)+(a+d)+(a+2d)+\ldots+(a+(n-1)d)$$

$$(a+(n-1)d)+(a+(n-2)d)+\ldots\ldots+a$$

$$(2a+(n-1)d)+(2a+(n-1)d)+\ldots+(2a+(n-1)d)$$

$$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$$

Jun 2-9:53 AM

Deriving the formulae for arithmetic series:

$$S_n = t_1 + t_2 + t_3 + \ldots + t_{n-2} + t_{n-1} + t_n$$

Recall: $t_n = a + (n - 1)d$, thus

$$S_n = t_1 + (t_1 + d) + (t_1 + 2d) + \ldots + (t_n - 2d) + (t_n - d) + t_n \quad ①$$

Re-arrange the order of the terms in S_n

$$S_n = t_n + (t_n - d) + (t_n - 2d) + \ldots + (t_1 + 2d) + (t_1 + d) + t_1 \quad ②$$

Add equation 1 and equation 2 so that the "d's" eliminate.

$$2S_n = [t_1 + t_n] + [t_1 + t_n] + [t_1 + t_n] + \ldots + [t_1 + t_n] + [t_1 + t_n] + [t_1 + t_n]$$

There are n of these $[t_1 + t_n]$ terms

$$2S_n = n[t_1 + t_n]$$

$S_n = \frac{n}{2}[t_1 + t_n]$ This is one of the formulae

sub. in $t_n = a + (n - 1)d$

$$S_n = \frac{n}{2}[a + a + (n-1)d]$$

$S_n = \frac{n}{2}[2a + (n-1)d]$ This is the other formulae

May 31-12:12 PM

Definitions:

3) A <u>geometric series</u> is the sum of the terms in a geometric sequence.

The sum can be found using the following formula:

$$s_n = \frac{a\left(r^n - 1\right)}{r - 1}, r \neq 1$$

Recall: $t_n = ar^{n-1}$

May 30-2:30 PM

Ex 3) For the geometic series with a = 7 and r = 2, determine S_9

$$S_n = \frac{a(r^n - 1)}{r - 1}, r \neq 1$$

$$S_9 = \frac{7(2^9 - 1)}{2 - 1}$$

$$S_9 = 7(512 - 1)$$

$$= 7(511)$$

$$= 3577$$

May 30-2:33 PM

Ex 4) Determine the sum of the series

15 + 45 + 135 + 405 + ... + 32 805

S_n, $n=?$ $\quad a=15 \quad r=3$

$t_n = ar^{n-1}$

$\frac{32805}{15} = \frac{15(3)^{n-1}}{15}$

$2187 = 3^{n-1}$

$3^7 = 3^{n-1}$

$n-1=7$

$n=8$

$S_n = \frac{a(r^n-1)}{r-1}$

$S_8 = \frac{15(3^8-1)}{3-1}$

$= \frac{15(6561-1)}{2}$

$= 49200$

May 30-2:33 PM

$S_n = a + \cancel{ar} + \cancel{ar^2} + \ldots + \cancel{ar^{n-1}}$

$(a + ar + ar^2 + \ldots + ar^{n-1}) \times r$

$rS_n = \cancel{ar} + \cancel{ar^2} + \ldots + \cancel{ar^{n-1}} + ar^n$

$rS_n - S_n = ar^n - a$

$S_n(r-1) = a(r^n-1)$

$S_n = \frac{a(r^n-1)}{r-1}$

Jun 2-10:09 AM

Deriving the formula for geometric series:

$$S_n = t_1 + t_2 + t_3 + ... + t_{n-2} + t_{n-1} + t_n$$

Recall: $t_n = ar^{n-1}$, thus

$$S_n = a + ar + ar^2 + ... + ar^{n-2} + ar^{n-1} \quad ①$$

Multiply equation 1 by *r*

$$rS_n = ar + ar^2 + ar^3 + ... + ar^{n-1} + ar^n \quad ②$$

Subtract the two equations so that most of the terms eliminate.

$$S_n - rS_n = a - ar^n$$

Factor S_n on the left hand side and
factor *a* on the right hand side

$$S_n(1-r) = a(1-r^n)$$

$$S_n = \frac{a(1-r^n)}{(1-r)}, \quad r \neq 1$$

To obtain the same equation as in our notes multiply both numerator and denominator by -1.

$$S_n = \frac{a(r^n-1)}{(r-1)}, \quad r \neq 1$$

This is the formula

May 31-12:12 PM

Assigned Work: P. 469 #2ac, 3ac, 4a, 7a, 20
P. 476 #1ac, 2ac, 3ac, 4, 12a, 17

May 27-3:05 PM