

Solving Linear Trigonometric equations

To solve a trigonometric equation means to find the angle value(s) that satisfy the given equation.

recall: solving linear equations

Solve for x:

$$2x + 3 = 4x - 5$$

$$\begin{array}{r} -2x = -8 \\ \underline{-2} \quad \underline{-2} \\ x = 4 \end{array}$$

Apr 19-9:13 PM

Solving Linear Trigonometric equations

May 5/
2011

To solve a trigonometric equation means to find the angle value(s) that satisfy the given equation.

Steps:

- ① the equation should involve only one trigonometric ratio
- ② isolate the trigonometric ratio
- ③ solve for the related acute angle (using positive ratio)
- ④ use the actual sign of the ratio to determine the quadrant(s) for your answer [CAST]
- ⑤ determine the angles, within your chosen quadrants, using the related acute angle

May 4-12:46 PM

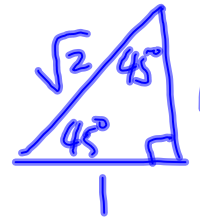
Ex: 1) Solve each equation for $0^\circ < \theta \leq 360^\circ$.

a) $\sin \theta = \frac{\sqrt{2}}{2}$

① ✓ ② ✓ ③ solve for RAA

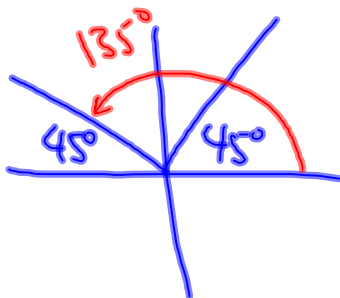
$\sin \theta = \frac{1}{\sqrt{2}}$

$\theta = 45^\circ$ or 135°



RAA = 45°

⑤



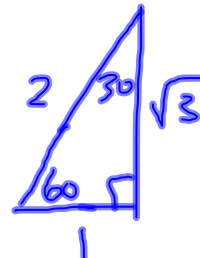
④ use CAST



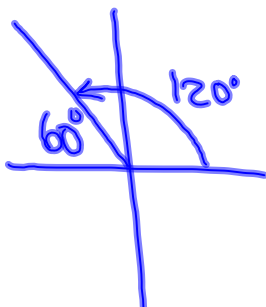
May 4-12:56 PM

b) $\cos \theta = \frac{-1}{2}$

$\theta = 120^\circ$ or 240°



RAA = 60°



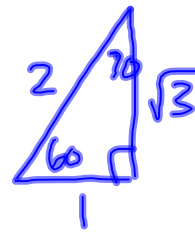
May 4-12:54 PM

c) $\sin \theta + \sqrt{3} = -\sin \theta$

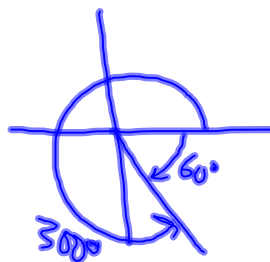
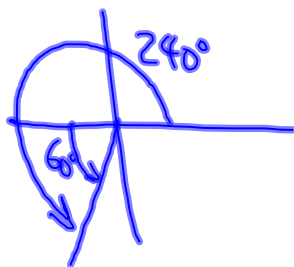
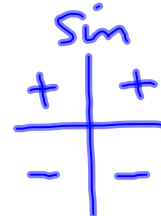
$2\sin \theta = -\sqrt{3}$

$\sin \theta = -\frac{\sqrt{3}}{2}$

$\theta = 240^\circ$ or 300°



RAA = 60°



May 4-12:59 PM

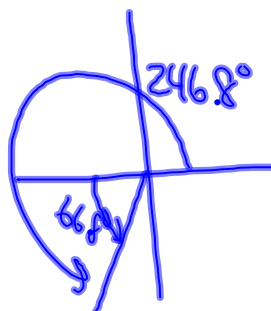
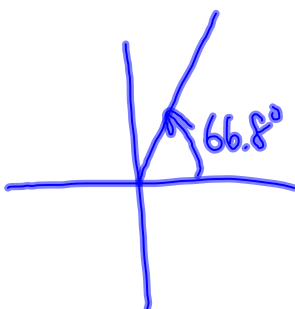
d) $5 \tan \theta = 2 \tan \theta + 7$

$3 \tan \theta = 7$

$\tan \theta = \frac{7}{3} \rightarrow$ no exact value.

$\theta = 66.8^\circ$ or 246.8°

RAA = $\tan^{-1}\left(\frac{7}{3}\right)$



RAA = 66.8°



May 4-1:00 PM

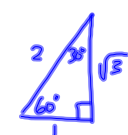
e) $\cos(2\theta) = \frac{\sqrt{3}}{2}$

let $\alpha = 2\theta$

$\cos \alpha = \frac{\sqrt{3}}{2}$

$\alpha = 30^\circ$ or $\alpha = 330^\circ$
but $\alpha = 2\theta$

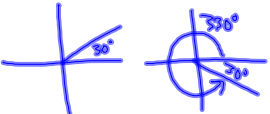
$2\theta = 30^\circ$ or $2\theta = 330^\circ$
 $\theta = 15^\circ$ or $\theta = 165^\circ$



RAA = 30°

S (A) C
T (C)

$\frac{\cos}{-}$
 $\frac{+}{+}$



there are some complications:

- asked to solve for $0^\circ < \theta \leq 360^\circ$
for 2θ $0^\circ < 2\theta \leq 720^\circ$

also have to consider coterminal angles
for 30° and 330° that satisfy $0^\circ < 2\theta \leq 720^\circ$

$\frac{+360^\circ}{390^\circ}$ $\frac{+360^\circ}{690^\circ}$

$\therefore 2\theta = 390^\circ$ or $2\theta = 690^\circ$
 $\theta = 195^\circ$ $\theta = 345^\circ$

$\therefore \theta = 15^\circ$ or 165° or 195° or 345°

May 4-1:00 PM

f) $3 \sin 2\theta + 3 = 5$

May 4-1:01 PM

Assigned Work:

p.408 # 1ace, 2adf, 5ab, 14agi

May 4-1:02 PM

p.408

$$\# 5(b) \quad 1 + \sin x = 4 \sin x$$

$$1 = 3 \sin x$$

$$\sin x = \frac{1}{3}$$

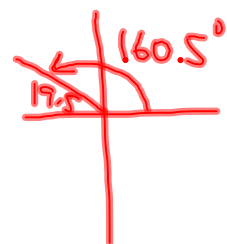
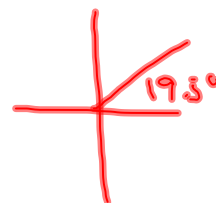
$$x = 19.5^\circ \text{ or } 160.5^\circ$$

$$RAA = \sin^{-1}\left(\frac{1}{3}\right)$$

$$\approx 19.5^\circ$$

S	A
T	C

sin	
+	+
-	-



May 6-9:10 AM

$$1(d) \sqrt{2} \sin x + 1 = 0$$

$$\sqrt{2} \sin x = -1$$

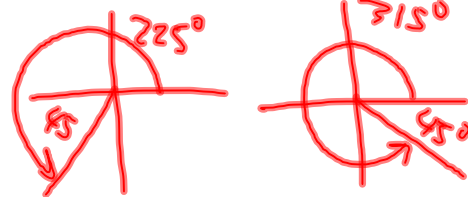
$$\sin x = -\frac{1}{\sqrt{2}}$$

$$x = 225^\circ \text{ or } 315^\circ$$

$$\begin{aligned} & \frac{5\pi}{4} \\ & = \frac{5(180^\circ)}{4} \\ & = 225^\circ \end{aligned}$$

$$\pi = 180^\circ$$

$$RAA = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$



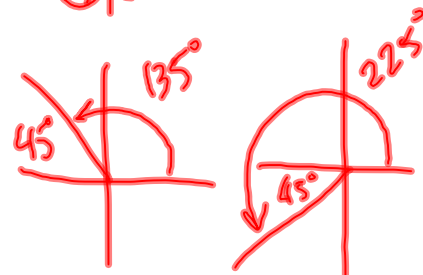
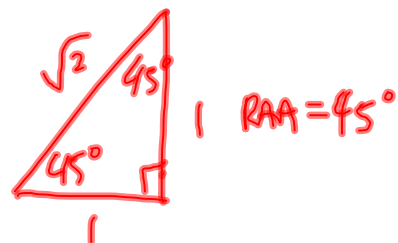
May 6-9:14 AM

$$2(d) \sqrt{2} \cos x + 1 = 0$$

$$\sqrt{2} \cos x = -1$$

$$\cos x = -\frac{1}{\sqrt{2}}$$

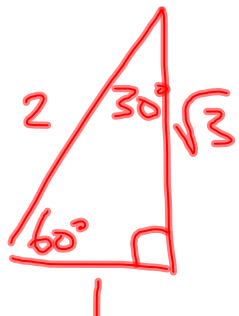
$$x = 135^\circ \text{ or } 225^\circ$$



May 6-9:18 AM

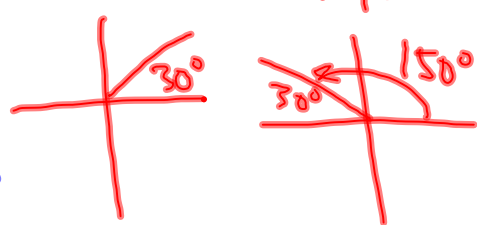
14(i) $2\sin(0.5x) = 1$
 $\sin(0.5x) = \frac{1}{2}$
 let $y = 0.5x$
 $\sin y = \frac{1}{2}$
 $y = 30^\circ$ or 150°
 $0.5x = 30^\circ$ or $0.5x = 150^\circ$
 $x = 60^\circ$ or $x = 300^\circ$

RAA = $\sin^{-1}\left(\frac{1}{2}\right)$



RAA = 30°

S	A
T	C

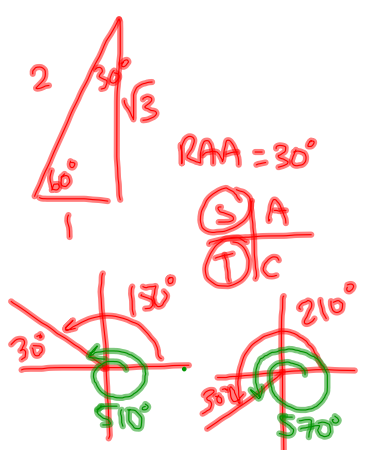


May 6-9:20 AM

14(g) $2\cos(2x) = -\sqrt{3}$
 $\cos(2x) = -\frac{\sqrt{3}}{2}$
 let $y = 2x$
 $\cos y = -\frac{\sqrt{3}}{2}$
 $y = 150^\circ$ or $y = 210^\circ$
 $2x = 150^\circ$ or $2x = 210^\circ$
 $x = 75^\circ$ or $x = 105^\circ$
 but $0^\circ \leq x \leq 360^\circ \rightarrow 0^\circ \leq 2x \leq 720^\circ$
 must consider coterminal angles

RAA = 30°

S	A
T	C



$2x = 150^\circ + 360^\circ$ $2x = 210^\circ + 360^\circ$
 $2x = 510^\circ$ $2x = 570^\circ$
 $x = 255^\circ$ $x = 285^\circ$

$x = 75^\circ, 105^\circ, 255^\circ, 285^\circ$

May 6-9:24 AM