Solving non-Linear Trigonometric Equations

May 6/2011

Here we are still looking for the angle value(s) that satisfy the given equation; the equations will be more complex and will require a few more steps to solve.

recall: solving non-linear equations

Solve for x:

a) $x + xy = 0$

$x(1+y)=0$

$x=0$ or $1+y=0$

$y=-1$

b) $x^2 + 3x - 4 = 0$

$(x+4)(x-1)=0$

$x+4=0$ or $x-1=0$

$x=-4$ $x=1$

Apr 19-9:13 PM

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Steps:

- the equation should involve only one trigonometric ratio
- move all the terms to one side of the equal sign so that the equation equals zero
- factor and set each factor to zero, or use the quadratic formula, to solve for the trigonometric ratio
- solve for the related acute angle for each of the ratios
- use the sign of the ratio to determine what quadrant(s) your answer should be in [CAST]
- determine the angles, within your chosen quadrants, using the related acute angle

May 4-12:46 PM

Ex: 1) Solve each equation for $0° < \theta \le 360°$.

a) $\cos\theta = 2\sin\theta\cos\theta$

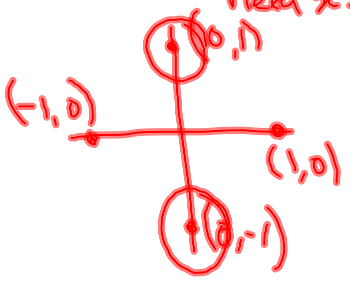
$0 = 2\sin\theta\cos\theta - \cos\theta$

$0 = \cos\theta(2\sin\theta - 1)$

$\cos\theta = 0$

$\cos\theta = \frac{x}{r}$

need $x = 0$



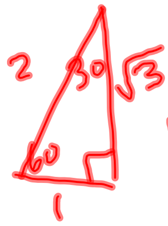
$\theta = 90°$ or $\theta = 270°$

$2\sin\theta - 1 = 0$

$2\sin\theta = 1$

$\sin\theta = \frac{1}{2}$

$\theta = 30°$ or $150°$



RAA = 30°

S A
T C

Q1: 30°

Q2: 150°

May 4-12:56 PM

b) $4\cos^2\theta = 3$ $\longrightarrow$ $4\cos^2\theta - 3 = 0$

$\cos^2\theta = \frac{3}{4}$

$4\cos^2\theta - (\sqrt{3})^2 = 0$

$\cos\theta = \pm\sqrt{\frac{3}{4}}$

$(2\cos\theta - \sqrt{3})(2\cos\theta + \sqrt{3}) = 0$

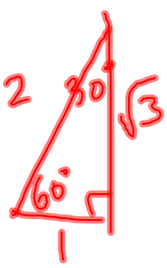
$\cos\theta = \pm\frac{\sqrt{3}}{2}$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\theta = 30°, 330°$

$\cos\theta = -\frac{\sqrt{3}}{2}$

$\theta = 150°, 210°$



RAA = 30°

-S | A+
-T | C+

May 4-12:54 PM

c) $\sin^2\theta + 4\sin\theta = 5$

$x^2 + 4x = 5$

$\sin^2\theta + 4\sin\theta - 5 = 0$

$x^2 + 4x - 5 = 0$

$(\sin\theta - 1)(\sin\theta + 5) = 0$

$(x-1)(x+5) = 0$

$\sin\theta = 1$ or $\sin\theta = -5$

no solution

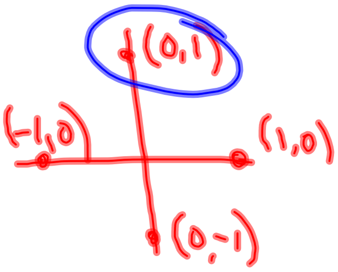
$\sin\theta = \frac{y}{r}$

$\frac{y}{1} = 1$

$\rightarrow y = 1$

$\sin\theta = \frac{y}{r}$

expect $\sin\theta \leq 1$



$\theta = 90°$

May 4-12:59 PM

d) $-3\sin^2\theta - 5\cos\theta + 2 = 0$

$\sin^2\theta = 1 - \cos^2\theta$

$-3(1-\cos^2\theta) - 5\cos\theta + 2 = 0$

$-3 + 3\cos^2\theta - 5\cos\theta + 2 = 0$

let $x = \cos\theta$

$3\cos^2\theta - 5\cos\theta - 1 = 0$

$3x^2 - 5x - 1 = 0$

cannot be factored

$\cos\theta =$ ____ $\cos\theta =$ ____

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

⋮

$x =$ ____ or ____

- determine RAA
- CAST
- state θ

May 4-1:00 PM

e) $3\sin\theta - 2\cos\theta = 0$

$$\frac{3\sin\theta}{\cos\theta} = \frac{2\cos\theta}{\cos\theta}$$

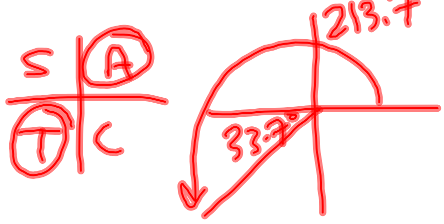
$$3\tan\theta = 2$$

$$\tan\theta = \frac{2}{3}$$

$\theta = 33.7^\circ$ or $\theta = 213.7^\circ$

$RAA = \tan^{-1}\left(\frac{2}{3}\right)$

$RAA \doteq 33.7^\circ$



May 4-1:00 PM

f) $3\sin 2\theta + 3 = 5$

$3\sin 2\theta = 2$

$\sin 2\theta = \frac{2}{3}$

$RAA = \sin^{-1}\left(\frac{2}{3}\right)$

$RAA \doteq 41.8^\circ$

$2\theta \doteq 41.8^\circ$ or $2\theta \doteq 138.2^\circ$

$\theta \doteq 20.9^\circ$ $\quad \theta = 69.1^\circ$

but $0^\circ \le \theta \le 360^\circ \rightarrow 0^\circ \le 2\theta \le 720^\circ$

$2\theta \doteq 41.8^\circ + 360^\circ$ $\quad 2\theta = 138.2^\circ + 360^\circ$

$2\theta = 401.8^\circ$ $\quad 2\theta = 498.2^\circ$

$\theta = 200.9^\circ$ $\quad \theta = 249.1^\circ$

$\therefore \theta = 20.9^\circ, 69.1^\circ, 200.9^\circ,$ or 249.1°

May 4-1:01 PM

Assigned Work: Pg. 408 #3abcde, 11

May 4-1:02 PM