Review of Exponent Laws

Apr.7/2011

Exercises: p.9 #1-9(odd), 12, 16

Recall:
A **power** is a product of identical factors and consists of two parts: a **base** and an **exponent**.

$$2^3$$

base = the identical factor
exponent = how many factors there are altogether

expanded form:

Evaluate $2^3 = 2\cdot 2\cdot 2$
$= 8$

Example 1: Evaluate.

a) $(-3)^3$
$=(-3)(-3)(-3)$
$=-27$

b) 3^3
$= 3\cdot 3\cdot 3$
$= 27$

c) -1^4
$= -(1^4)$
$= -(1)$
$= -1$

d) $(-1)^4$
$=(-1)(-1)(-1)(-1)$
$= 1$

e) 3^4
$= 3\cdot 3\cdot 3\cdot 3$
$= 81$

f) $\left(\frac{4}{3}\right)^2 = \left(\frac{4}{3}\right)\left(\frac{4}{3}\right)$
$= \frac{16}{9}$
$= \frac{4^2}{3^2}$

Rule #1: Multiplication of Powers with the same base

to investigate the rule let's look at a specific example and go through the process of expanding before simplifying.

$$\left(3^{1}\right)\left(3^{2}\right) = (3)(3 \cdot 3)$$
$$= 3^{3}$$
$$= 3^{1+2}$$

The Rule: $(a^{x})(a^{y}) = a^{x+y}$

In words: when multiplying powers with the same base, add exponents.

Rule #2: Division of Powers with the same base

$$3^{1} \div 3^{2} = \frac{3^{1}}{3^{2}}$$
$$= \frac{\cancel{3}^{1}}{\cancel{3}_{1} \cdot 3}$$
$$= \frac{1}{3}$$
$$=$$

The Rule: $a^{x} \div a^{y} = \dfrac{a^{x}}{a^{y}}$

$$= a^{x-y}, \ a \neq 0$$

In words: when dividing powers with the same base, subtract exponents.

Rule #3: Power of a Power

$$\left(3^2\right)^4 = (3^2)(3^2)(3^2)(3^2)$$
$$= (3 \cdot 3)(3 \cdot 3)(3 \cdot 3)(3 \cdot 3)$$
$$= 3^8$$
$$= 3^{2 \cdot 4}$$

The Rule: $\left(a^x\right)^y = a^{xy}$

In words: when having a power to an exponent, multiply the exponents.

Rule #4: Identity Rule

What exponent does not change the value of a power?

The Rule: $a^1 = a$

In words: anything to the exponent of 1 is equal to itself.

Rule #5: Zero Exponent

Look at the expanded form of powers and find a pattern:

$2^3 =$ 8

÷ 2

$2^2 =$ 4

÷ 2

$2^1 =$ 2

÷ 2

then $2^0 =$ 1

The Rule: $a^0 = 1,\ a \neq 0$

In words: anything to the exponent of zero is 1.
This is because an exponent of zero means you are dividing the base by itself.

Rule #6: Negative Exponent

Continue the pattern from the previous rule:

$2^0 =$ 1

÷ 2

$2^{-1} = \frac{1}{2^1}$

÷ 2

$2^{-2} = \frac{1}{4}$

$= \frac{1}{2^2}$

The Rule: $a^{-x} = \left(\frac{1}{a}\right)^x,\ a \neq 0$

In words: a negative exponent requires you to find the reciprocal of the base.

Rule #7: Distributive Rule (for powers with different bases!)

$$(7^2 \cdot 2^5)^3 = (7^2 \cdot 2^5)(7^2 \cdot 2^5)(7^2 \cdot 2^5)$$
$$= (7^2)^3 (2^5)^3$$
$$= 7^6 \cdot 2^{15}$$

$$\left(\frac{7^2}{2^5}\right)^3 = \left(\frac{7^2}{2^5}\right)\left(\frac{7^2}{2^5}\right)\left(\frac{7^2}{2^5}\right)$$
$$= \frac{(7^2)^3}{(2^5)^3}$$
$$= \frac{7^6}{2^{15}}$$

The Rule: $(ab)^x = (a^x)(b^x)$ $\qquad \left(\frac{a}{b}\right)^x = \frac{a^x}{b^x},\ b \neq 0$

$$(a^m b^n)^p = a^{mp} b^{np} \qquad \left(\frac{a^m}{b^n}\right)^p = \frac{a^{mp}}{b^{np}}$$

In words: when having more than one power to an exponent, distribute the exponent to each of the powers.

<u>Example 2</u>: Simplify. Express your final answer with positive exponents.

a) $(4^{-6})(4^4)$
$= 4^{-2}$
$= \frac{1}{4^2} \rightarrow = \frac{1}{16}$

b) $\frac{(-3)^2}{(-3)^{-3}} = (-3)^{2-(-3)}$
$= (-3)^5$
$= -243$

c) $(5^{-2} \times 5^4)^{-2}$
$= (5^2)^{-2}$
$= \frac{1}{5^4} \rightarrow = \frac{1}{625}$

d) $(x^{-3} y^5)^{-3}$
$= x^9 y^{-15}$
$= \frac{x^9}{y^{15}}$

e) $(3a^2b)(-2a^3b^4)$
$= -6a^5b^5$

f) $(-a^5b)^{-2}(-ab^{-2})^2$
$= \frac{(-ab^{-2})^2}{(-a^5b)^2}$
$= \frac{a^2b^{-4}}{a^7b^2}$
$= a^{-5}b^{-6}$
$= \frac{1}{a^5b^6}$

The exponent laws also work if you have polynomials instead of numbers as exponents.

Ex.3: Simplify

a) $(x^3)^{2a+4}$

$= x^{3(2a+4)}$

$= x^{6a+12}$

b) $(x^{a+5})(x^{3a+1})$

$= x^{(a+5)+(3a+1)}$

$= x^{4a+6}$

c) $(x^{4m - 3n}) \div (x^{m + 5n})$

$= x^{(4m-3n)-(m+5n)}$

$= x^{3m-8n}$

d) $x^y(x^{y+1})^{y+2}(1/x)^{6y}$

$= x^y (x^{y+1})^{y+2} x^{-6y}$

$= x^y x^{(y+1)(y+2)} x^{-6y}$

$= x^y x^{y^2+3y+2} x^{-6y}$

$= x^{y^2-2y+2}$

Assigned Work:

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