Exponential Applications (part 2)
Doubling Period & Half-Life

See also
p. 19-23

Apr 14-9:08 AM

Exponential Applications (part 2)
Doubling Period & Half-Life

Summary of Exponential Functions:

$$y = a(b)^x + q,\ b > 0,\ b \neq 1$$

where:

- a is the scale factor
- q is the lower or upper bound (asymptote)
- $a + q$ is the initial value/amount (y-intercept)
- x is the number of periods/cycles, elapsed time, etc.
- y is the measured value after x

Exponential Growth: b > 1   Exponential Decay: 0 < b < 1

Apr 11-10:11 AM

Some relations occur so frequently that we have created special equations for them. It is not necessary to use these equations, but the benefit to using them is that the common ratio is known and does not need to be calculated.

Apr 12-9:19 PM

Doubling Period: The time required for a quantity to grow to twice its original amount (b = 2).

The number of periods, x, becomes $\frac{t}{D}$

where *t* is the elapsed time, and *D* is the doubling period (i.e., the amount of time required to double the amount).

$y = a(b)^x + q$ becomes $y = a(2)^{\frac{t}{D}} + q$

Apr 13-11:12 PM

Ex.1 The number of a certain bacteria doubles every 4 hours. The initial population is 36.

(a) Determine an exponential model for the number of bacteria after t hours using $y = a(2)^{\frac{t}{D}} + \cancel{q} \rightarrow 0$

$a = 36$

$D = 4$

$y = 36(2)^{t/4}$

(b) Determine the number of bacteria after 8 hours.

$t = 8, \; y = 36(2)^{8/4}$

$= 36(2)^2$

$= 36(4)$

$= 144$

$\therefore$ the population of the bacteria after 8 hours is 144

Apr 13-11:12 PM

Ex.1 The number of a certain bacteria doubles every 4 hours. The initial population is 36.

(c) Determine an exponential model for the number of bacteria after t hours using $y = a(b)^x + q$

$\left.\begin{matrix} q = 0 \\ a = 36 \end{matrix}\right\} \; y = 36(b)^x$

when $t = 4, y = 72$

Sub into model

$72 = 36(b)^4$

$(2)^{1/4} = (b^4)^{1/4}$

$1.189 \doteq b$

$y = 36(1.189)^x$

(c) Determine the number of bacteria after 8 hours.

$t = 8, \; y = 36(1.189)^8$

$y \doteq 143.8$

essentially the same as $y = 36(2)^{t/4}$

Apr 13-11:12 PM

<u>Half-Life</u>: The time required for a material to decay (or be reduced) to one-half of its original quantity (b = $\frac{1}{2}$).

The number of periods, x, becomes $\dfrac{t}{h}$

where *t* is the elapsed time, and *h* is the half-life (i.e., the amount of time required to reduce the amount by half).

$y = a(b)^x + q$ becomes $y = a(\frac{1}{2})^{\frac{t}{h}} + q$

usually 0

Apr 12-9:20 PM

Ex.2 Archaeologists use the radioactive decay of carbon-14 to estimate the age of relics containing carbon. The half-life of carbon is 5370 years.

(a) Determine an exponential model for the amount of ^{14}C present after *t* years (as compared to ^{12}C) using

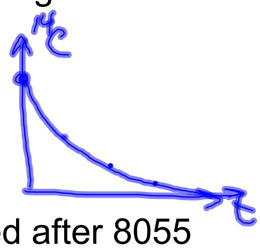
$y = a(\frac{1}{2})^{\frac{t}{h}} + q$

$h = 5370$

$q = 0$

$a = 100$

$y = 100\left(\frac{1}{2}\right)^{t/5370}$



(b) Determine the percentage of ^{14}C expected after 8055 years.

$t = 8055, \quad y = 100\left(\frac{1}{2}\right)^{8055/5370}$

$y \doteq 35.35$

$\therefore$ after 8055 years, expect 35% of ^{14}C
C-14

Apr 13-11:19 PM

Ex.2 Archaeologists use the radioactive decay of carbon-14 to estimate the age of relics containing carbon. The half-life of carbon is 5370 years.

(c) Determine an exponential model for the amount of ^{14}C present after *t* years (as compared to ^{12}C) using

$y = a(b)^x + q$

$q = 0$, $a = 100$ $\Big\}$ $y = 100(b)^x$

$b = ?$

when $x = 5370$, $y = 50$ $\Big\}$ $50 = 100(b)^{5370}$

$(0.5)^{\frac{1}{5370}} = (b^{5370})^{\frac{1}{5370}}$

$b = \sqrt[5370]{0.5}$

$b = 0.99987093$

$y = 100(0.99987093)^x$

$(x^2)^{\frac{1}{2}} = (4)^{\frac{1}{2}}$

$x^1 = 2$

(d) Determine the percentage of ^{14}C expected after 8055 years.

$y = 100(0.99987093)^{8055}$

$y \doteq 35.35$

Apr 13-11:19 PM

Exercises:

handout # 1-9

Apr 6-9:18 PM

5. $h = 2$ years

$m_0 = 5.0$ kg (initial amount)

(b) $t = 18$ months

$$m(t) = m_0 \left(\frac{1}{2}\right)^{t/h}$$

$$= 5.0 \left(\frac{1}{2}\right)^{t/2}$$ ← years.

$t = 1.5$ years OR $m(t) = 5.0 \left(\frac{1}{2}\right)^{t/24}$ ← months

$$m(1.5) = 5\left(\frac{1}{2}\right)^{1.5/2}$$

$m(1.5) = 2.973$ kg. ∴ —

Apr 15-11:09 AM

8.(a) $V(t) = V_0 (0.8)^t$

(b) $V_0 = 38900$

$$V(6) = 38900 (0.8)^6$$

Apr 15-11:13 AM

9. $V(t) = a(b)^t$

$V(0) = 3200$

t years, $V(\underset{\text{3 months}}{0.25}) = 3125$

$3200 = a(b)^0$

$3200 = a$

$3125 = a(b)^{0.25}$

$3125 = 3200(b)^{0.25}$

$(0.9765)^4 = (b^{0.25})^4$

$0.9092 = b$

$V(t) = 3200(0.9092)^t$

(b) 6 months from now is 9 months from 3200

$t = 0.75$

$V(0.75) = 3200(0.9092)^{0.75}$

Apr 15-11:17 AM

2(c)

$P(t) = 35000(1.02)^t$

$P(25) = 35000(1.02)^{25}$

Apr 15-11:24 AM