Feb24/2011

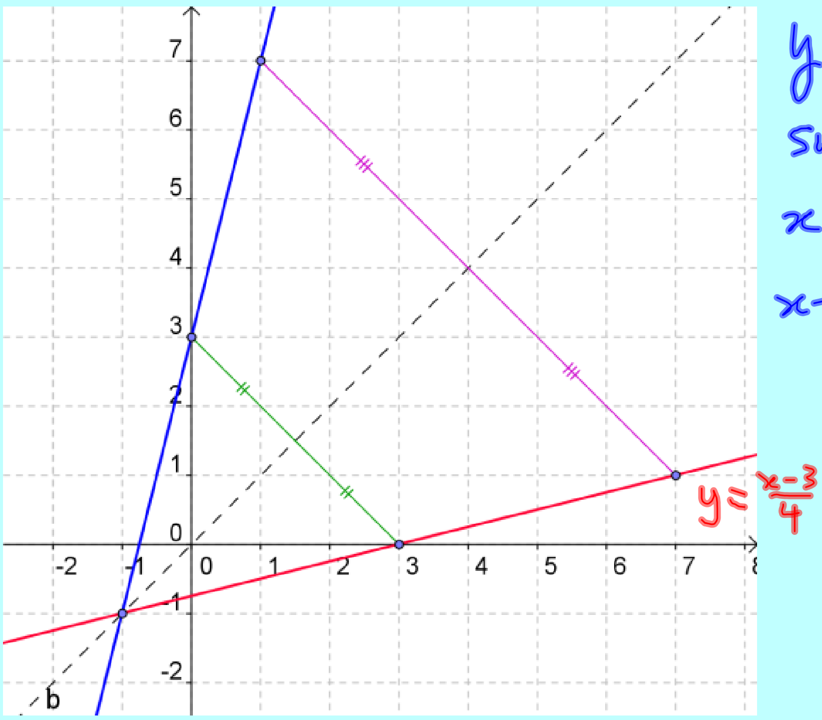
Restricting the Domain & The Radical Function

The inverse of a relation can be found by interchanging the domain and range of the relation.

Graphically, swapping the x- and y-values is equivalent to reflecting the relation across the line y = x.

Feb 22-9:25 PM

Find the inverse of $y = 4x + 3$ graphically



$y = 4x + 3$

swap x, y

$x = 4y + 3$

$x - 3 = 4y$

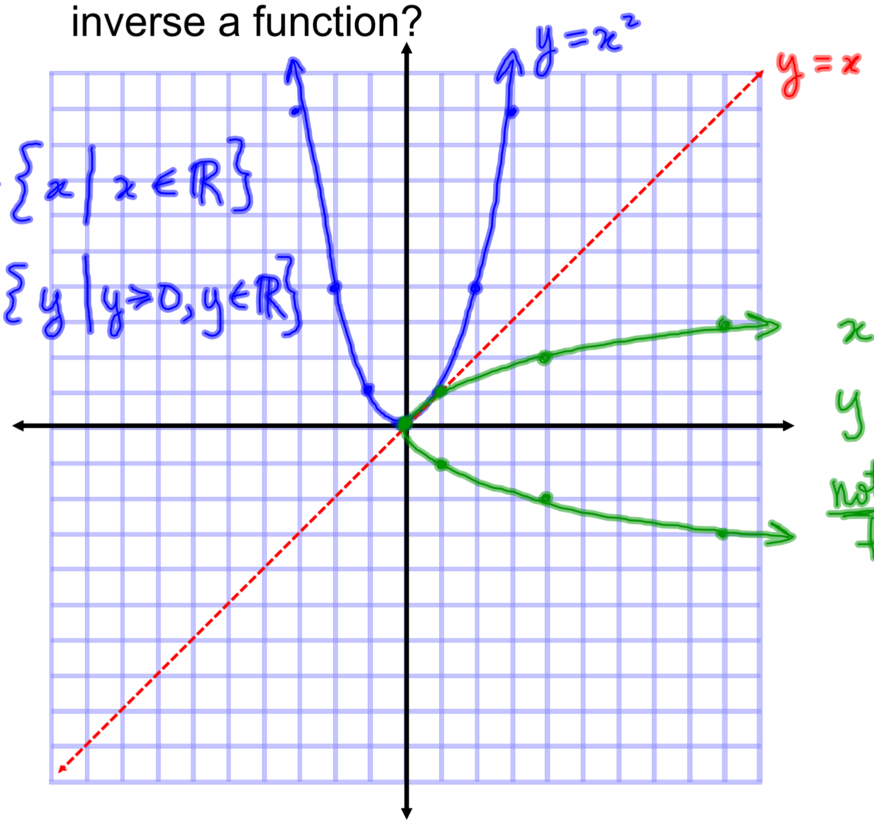
$y = \frac{x-3}{4}$

Feb 23-9:20 PM

Ex.1 Find the inverse of $y = x^2$ graphically. Is the inverse a function?

$D = \{x \mid x \in \mathbb{R}\}$

$R = \{y \mid y \geq 0, y \in \mathbb{R}\}$



$x = y^2$

$y = \pm\sqrt{x}$

not a function

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Algebraically, swap the x- and y-variables, then rearrange the new equation for y.

If the original equation is in function notation, first change the function to y.

Ex.2 Find the inverse of $f(x) = x^2$ algebraically.

Feb 22-10:21 PM

Recall: A <u>function</u> is a special type of relation where each element in the domain corresponds to a single value in the range.

?

If the inverse of the function, $f(x)$ is also a function, it is given the special designation of <u>inverse function</u>, $f^{-1}(x)$

Note: In the inverse notation, the "-1" is not an exponent!

For example:

$$x^{-1} = \frac{1}{x} \qquad f^{-1}(x) \neq \frac{1}{f(x)}$$

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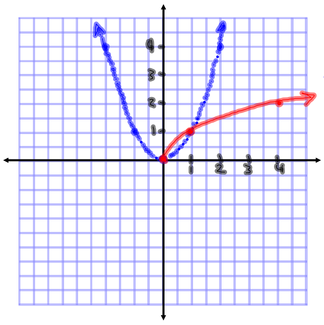
If the inverse of a function is not a function, it is still possible to create a function by <u>restricting the domain</u> of the original function.

Ex.3 Restrict the domain of $y = x^2$ so the inverse is also a function.



could restrict domain to

$x \geq 0$ or $x \leq 0$

the inverse is now a function

$y = x^2$

swap x, y

$x = y^2$

$y = \pm\sqrt{x}$

but $f(x) = x^2$ has $D = \{x \mid x \geq 0, x \in \mathbb{R}\}$

$R = \{y \mid y \geq 0, y \in \mathbb{R}\}$

for the inverse, domain & range are also swapped

$D = \{x \mid x \geq 0, x \in \mathbb{R}\}$

$R = \{y \mid y \geq 0, y \in \mathbb{R}\}$

$y = \pm\sqrt{x} \longrightarrow y = \sqrt{x}$

$f^{-1}(x) = \sqrt{x}$

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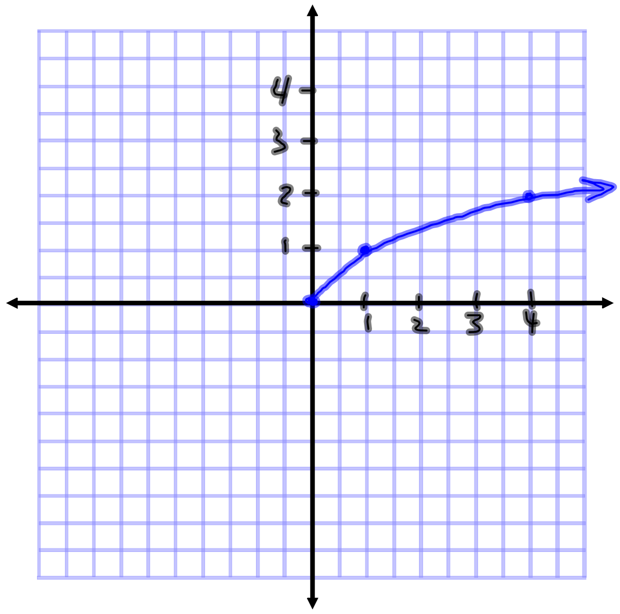
$f(x) = \sqrt{x}$ is the radical function.

Domain = $\{x \mid x \geq 0\}$ $x \in \mathbb{R}$ Range = $\{y \mid y \geq 0\}$ $y \in \mathbb{R}$



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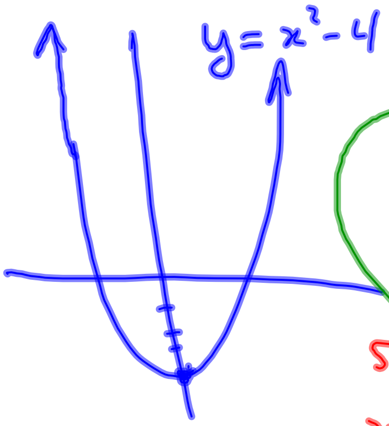
Ex.4 Find the inverse of $f(x) = x^2 - 1$
and restrict f(x) to make the inverse a function

Feb 22-10:36 PM

Assigned Work:

p.215 # ~~1-3~~, ~~5odd~~, ~~11~~, 13odd, 14odd, 16odd, 24



"domain is var x such that x is a real number"

$D = \{x \mid x \in \mathbb{R}\}$

$R = \{y \mid y \geq -4, y \in \mathbb{R}\}$

swap x, y

$x = y^2 - 4$

$y^2 = x + 4$

$y = \pm\sqrt{x+4}$

$D = \{x \mid x \geq -4, x \in \mathbb{R}\}$

$R = \{y \mid y \in \mathbb{R}\}$

Feb 10-10:23 PM

14. $f(x) = x^2$, $\underline{x \geq 0}$

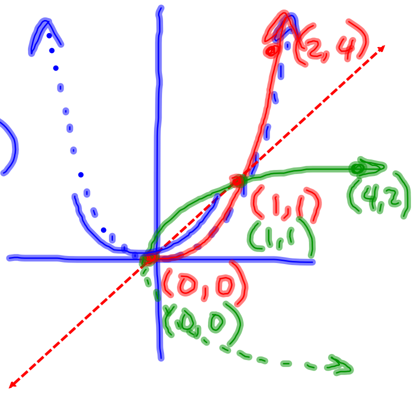
(a) $y = x^2$

swap x, y

$x = y^2$

$\pm\sqrt{x} = y$

(b)



(c) $D_f = \{x \mid x \in \mathbb{R}, x \geq 0\}$

$R_f = \{y \mid y \geq 0, y \in \mathbb{R}\}$

$D_{f^{-1}} = \{x \mid x \geq 0, x \in \mathbb{R}\}$

$R_{f^{-1}} = \{y \mid y \in \mathbb{R}, y \geq 0\}$

Feb 25-11:44 AM

$f(x) = x^2 - 2, \ x \geq 0$

(a) $y = x^2 - 2$

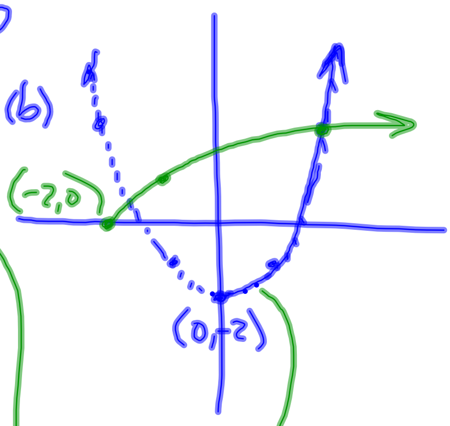
Swap x, y

$x = y^2 - 2$

$x + 2 = y^2$

$y = \pm\sqrt{x+2}$

(b)



(c) $D_f = \{x \mid x \in \mathbb{R}, x \geq 0\}$

$R_f = \{y \mid y \in \mathbb{R}, y \geq -2\}$

$D_{f^{-1}} = \{x \mid x \in \mathbb{R}, x \geq -2\}$

$R_{f^{-1}} = \{y \mid y \in \mathbb{R}, y \geq 0\}$

Feb 25-11:54 AM

16. (vi) $f(x) = (x-2)^2$

(a) $y = (x-2)^2$

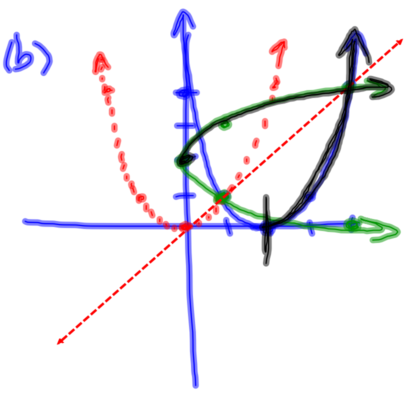
Swap x, y

$x = (y-2)^2$

$\pm\sqrt{x} = y - 2$

$y = 2 \pm \sqrt{x}$

(b)



$D_f = \{x \mid x \in \mathbb{R}, x \geq 2\}$

$R_f = \{y \mid y \in \mathbb{R}, y \geq 0\}$

$D_{f^{-1}} = \{x \mid x \in \mathbb{R}, x \geq 0\}$

$R_{f^{-1}} = \{y \mid y \in \mathbb{R}, y \geq 2\}$

Feb 25-12:01 PM

14 (iv) $f(x) = 3 - x^2,\ \underline{x \geq 0}$

$= -x^2 + 3$

(a) $y = 3 - x^2$

Swap x, y

$x = 3 - y^2$

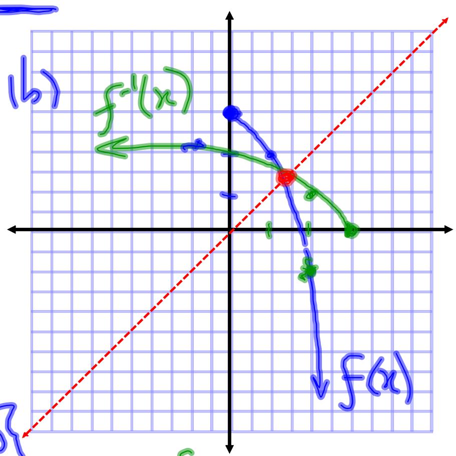
$y^2 = 3 - x$

$y = \pm\sqrt{3 - x}$

$D_f = \{x \mid x \in \mathbb{R},\ x \geq 0\}$

$R_f = \{y \mid y \in \mathbb{R},\ y \leq 3\}$

(b)



$D_{f^{-1}} = \{x \mid x \in \mathbb{R},\ x \leq 3\}$

$R_{f^{-1}} = \{y \mid y \in \mathbb{R},\ y \geq 0\}$

Feb 28-9:06 AM

13 (h) $y = 2x^2 - 1$

Swap x, y

$x = 2y^2 - 1$

$x + 1 = 2y^2$

$y^2 = \dfrac{x+1}{2}$

$y = \pm\sqrt{\dfrac{x+1}{2}}$

$y = \pm\dfrac{\sqrt{x+1}}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$y = \pm\dfrac{\sqrt{2}\sqrt{x+1}}{2}$

$y = \pm\dfrac{\sqrt{2(x+1)}}{2}$

$y = \pm\frac{1}{2}\sqrt{2(x+1)}$

Feb 28-9:16 AM

24. $T(d) = 35d + 20$

(a) $T = 35d + 20$

(b) $T - 20 = 35d$

$$d = \frac{T-20}{35}$$

$$d(T) = \frac{T-20}{35}$$

(c) $d(90) = \frac{90-20}{35}$

$= \frac{70}{35}$

$= 2$

$\therefore$ rocks 90°c at 2km

Feb 28-9:20 AM