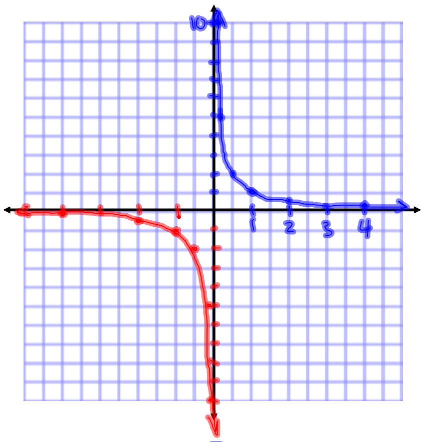
The Reciprocal Function

Feb 28/2011

Consider the relation $y = \frac{1}{x}$

Create a table of values and graph. Is it a function?
What are the domain and range?

x	y
4	0.25
2	0.5
1	1
0.5	2
0.2	5
0.1	10
0	undefined
-0.1	
-0.2	
-0.5	
⋮	



$D = \{x \mid x \in \mathbb{R}, x > 0, x < 0\}$

or $D_f = \{x \mid x \in \mathbb{R}, x \neq 0\}$ } D & R of $f(x)$

$R_f = \{y \mid y \in \mathbb{R}, y \neq 0\}$

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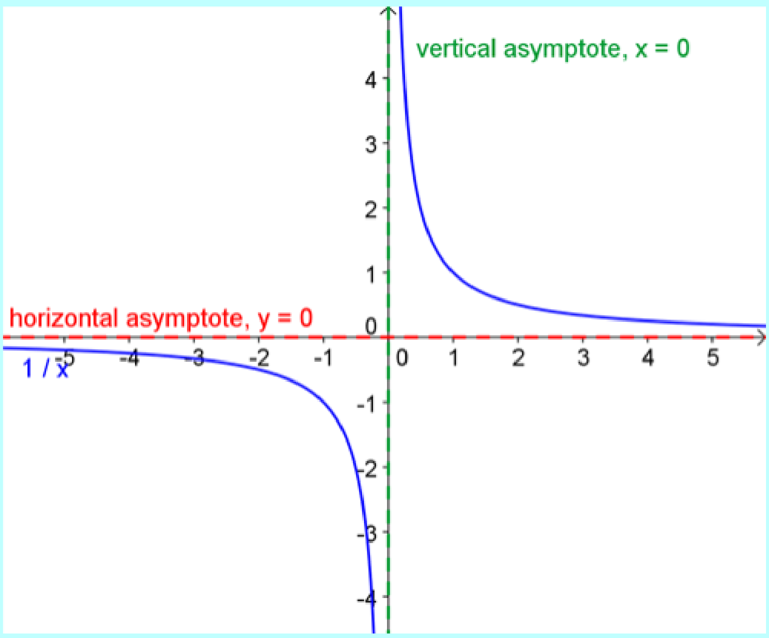
The reciprocal function, $f(x) = \frac{1}{x}$



$D = \{x \mid x \neq 0, x \in \mathbb{R}\}$ $R = \{y \mid y \neq 0, y \in \mathbb{R}\}$

Feb 23-9:20 PM

A line that a curve approaches, but never touches, is called an <u>asymptote</u>. The reciprocal function has two asymptotes:

Vertical Asymptote (VA): $x = 0$

Horizontal Asymptote (HA): $y = 0$

Note how these asymptotes correspond to the restrictions on the domain and range of the function.

$$D = \{x \mid x \neq 0, x \in \mathbb{R}\}$$

$$R = \{y \mid y \neq 0, y \in \mathbb{R}\}$$

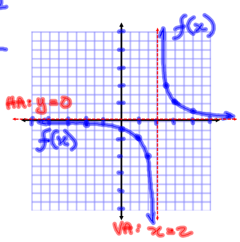
Feb 27-10:16 PM

Ex.1 Given $f(x) = \frac{1}{x-2}$

(a) graph the function
(b) state the domain and range
(c) determine the inverse
(d) graph the inverse
(e) restrict f(x) to make the inverse a function

(a) TOV around $x=2$

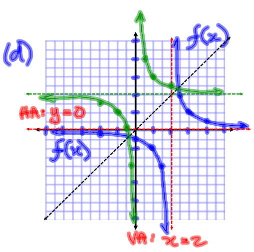
x	$f(x) = \frac{1}{x-2}$
-2	-0.25
0	-0.5
1	-1
1.5	-2
2	undef.
2.5	2
3	1
4	0.5
6	0.25



(b)
$D_f = \{x \mid x \in \mathbb{R}, x \neq 2\}$
$R_f = \{y \mid y \in \mathbb{R}, y \neq 0\}$

(c) $f(x) = \frac{1}{x-2}$
$y = \frac{1}{x-2}$
Swap x, y
$x = \frac{1}{y-2}$
$\cancel{x}(y-2) = \frac{1}{\cancel{x}}$
$y-2 = \frac{1}{x}$
$y = \frac{1}{x} + 2$
Could use TOV or reflect in $y=x$

(d)



(e) the inverse is a function
$D_{f^{-1}} = \{x \mid x \in \mathbb{R}, x \neq 0\}$
$R_{f^{-1}} = \{y \mid y \in \mathbb{R}, y \neq 2\}$

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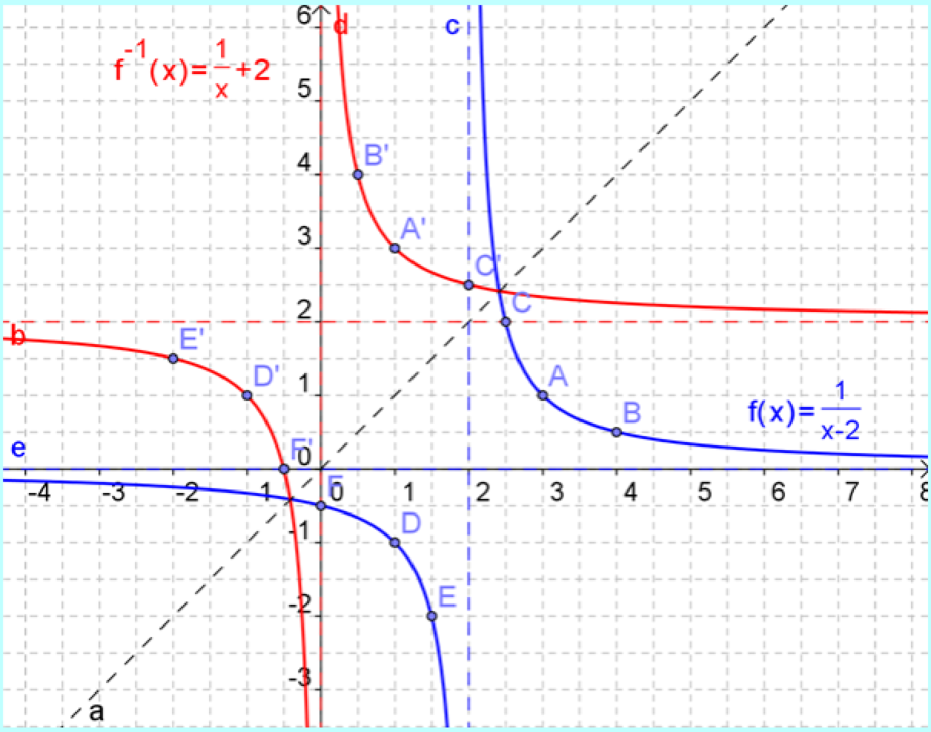

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Assigned Work:

Worksheet on interpreting function notation

$f(x)$	x	x^2	$\sqrt{x}$	$\frac{1}{x}$
$f(a)$	a	a^2	$\sqrt{a}$	$\frac{1}{a}$
$5\cdot f(a)$	$5a$	$5a^2$	$5\sqrt{a}$	$\frac{5}{a}$
$-f(a)$	$-a$	$-a^2$	$-\sqrt{a}$	$-\frac{1}{a}$
$\vdots$				
$f(-2a)$	$-2a$	$4a^2$	$\sqrt{-2a}$	$-\frac{1}{2a}$

Feb 10-10:23 PM

$$5 \cdot f[\underbrace{4(a-1)}_{x}]-3$$

$f(x)=x$:
$$5 \cdot [4(a-1)]-3$$
$$=5[4a-4]-3$$
$$=20a-20-3$$
$$=20a-23$$

$f(x)=x^2$:
$$5[4(a-1)]^2-3$$
$$=5 \cdot [4^2 \cdot (a-1)^2]-3$$
$$=5[16(a-1)^2]-3$$
$$=80(a-1)^2-3$$

$(a \cdot b)^2 = a^2 \cdot b^2$

$f(x)=\sqrt{x}$:
$$5\sqrt{4(a-1)}-3$$
$$=5\sqrt{4}\sqrt{a-1}-3$$
$$=10\sqrt{a-1}-3$$

$f(x)=\frac{1}{x}$:
$$5\left[\frac{1}{4(a-1)}\right]-3$$
$$=\frac{5}{4(a-1)}-3$$
$$=\frac{5}{4}\left(\frac{1}{a-1}\right)-3$$

Mar 1-10:28 AM

$$f\left(\frac{a}{2}\right)$$

$f(x)=x$: $f\left(\frac{a}{2}\right)=\frac{a}{2}$

$f(x)=x^2$:
$$f\left(\frac{a}{2}\right)=\left(\frac{a}{2}\right)^2$$
$$=\frac{a^2}{4}$$
$$=\frac{1}{4}a^2$$

scale factor $\frac{1}{4}$

$f(x)=\sqrt{x}$:
$$f\left(\frac{a}{2}\right)=\sqrt{\frac{a}{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$
$$=\frac{\sqrt{2a}}{2}$$
$$=\frac{1}{2}\sqrt{2a}$$

$f(x)=\frac{1}{x}$:
$$f\left(\frac{a}{2}\right)=\frac{1}{\frac{a}{2}}$$
$$=\frac{2}{a}$$
$$=2\left(\frac{1}{a}\right)$$

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$f(x)$	x	x^2	$\sqrt{x}$	$\frac{1}{x}$
$f(3a-6)$	$3a-6$	$(3a-6)^2$	$\sqrt{3a-6}$	$\frac{1}{3a-6}$

$$= [3(a-2)]^2$$
$$= 9(a-2)^2$$

$3\cdot f(4a-8)-2$ $\qquad 3\sqrt{4a-8}-2$

$$\frac{-3}{a-5}+2$$

$-3f(a-5)+2$ $\qquad = -3\left(\frac{1}{a-5}\right)+2$