Horizontal ~~Transformations~~ Scaling & Reflection

On Tuesday and Wednesday we reviewed the transformations that you studied in grade 10.

There are two transformations that we have not studied yet, but they should be studied.

Reflection along the y-axis: in function notation: $y = f(kx)$, $k < 0$.
i.e.: $y = f(-x)$

Horizontal Scaling: in function notation: $y = f(kx)$, $k > 0$. $y=f(kx)$

horizontal stretch: $0 < k < 1$

or horizontal compression: $k > 1$

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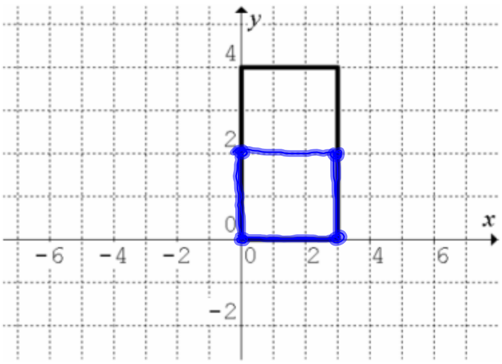
Lets compare the vertical and horizontal scaling using a rectangle (yes, we know it is not a function, but remember transformations can be applied to pretty much anything!)

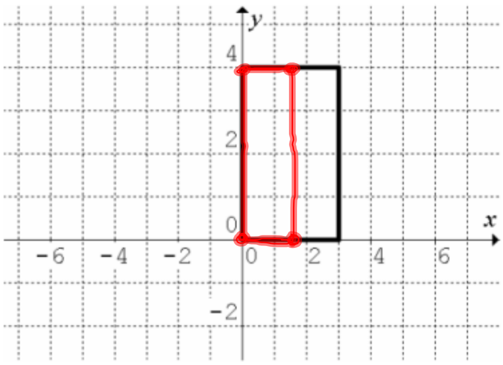
Ex. 1) Apply a vertical compression by a factor of 2 to the rectangle shown below.
(Use a table of values with the key points, if you want to!)



Function notation:

$y = \frac{1}{2}f(x)$

Apply a horizontal compression by a factor of 2 to the rectangle shown below.
(Use a table of values with the key points, if you want to!)



Function notation:

$y = f(2x)$

Do the transformed rectangles look the same? NO.

$y=f(x-1)$	x	$y=f(x)$	$y=2f(x)$
1? to go	0	0	0
2	1	1	2
3	2	4	8
4	3	9	18

shifted R by 1

$y=f(2x)$	x	$y=f(x)$
0	0	0
0.5	1	1
1	2	4
1.5	3	9

compressed by 2, divided by 2

$f(x)=x^2$

x	$y=f(x)$	$y=f(2x)$
0	0	0
$\frac{1}{2}$	$\frac{1}{4}$	1
1	1	4
$\frac{3}{2}$	$\frac{9}{4}$	9
2		
$\frac{5}{2}$		
3		

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Ex: 2) List the transformations to $y=f(x)$, in the appropriate order, defined by

a) $y=2f\left(\frac{1}{5}x\right)+3$

1. v. stretch by 2
2. h. scaling by 5
   h. stretch by 5
3. shift up by 3

b) $y=-f[2(x-3)]$

1. v. reflect
2. h. scaling by $\frac{1}{2}$
   h. comp by 2
3. shift right 3

c) $y=2f(-3x+15)$

need standard form

$y=af[k(x-p)]+q$

k must be factored

1. v. stretch 2
2. h. reflect
3. h. comp by 3
4. h. right 5

$y=2f(-3x+15)$

$y=2f[-3(x-5)]$

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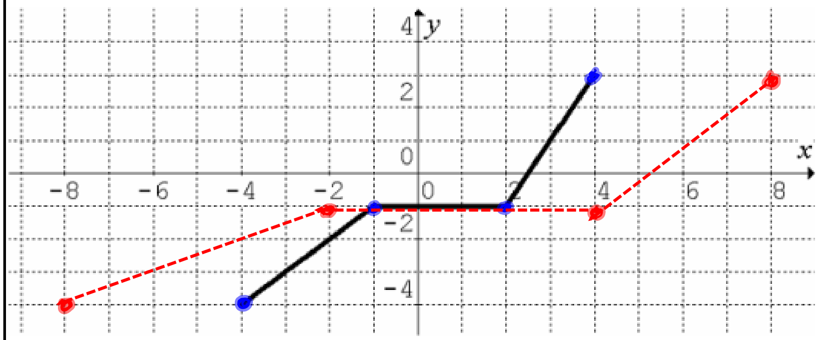
Ex: 3) The graph below shows the function $y = f(x)$. On the same grid sketch the function $y = f(0.5x)$

(Use a table of values with the key points, if you want to!)



$y = f(\underbrace{0.5}_{k=0.5}x)$

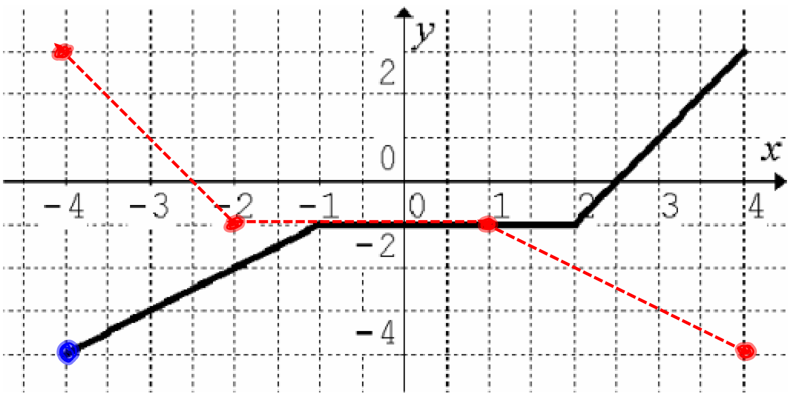
$(x, y) \rightarrow (2x, y)$

h. scaling is $\frac{1}{k} = \frac{1}{0.5} = 2$

h. stretch by 2

Ex: 4) The graph below shows the function $y = f(x)$. On the same grid sketch the function $y = f(-x)$

(Use a table of values with the key points, if you want to!)



$(x, y) \rightarrow (-x, y)$

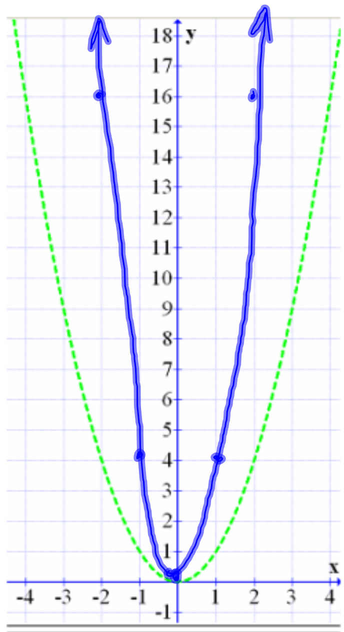
Why are these transformations not studied with parabolas?

Compare a vertical stretch by a factor of 4 to a horizontal compression by a factor of 2

Vertical stretch by a factor of 4: $y = 4x^2$ $\quad f(x) = x^2$

Table of values $\quad y = 4f(x)$

x	$y = x^2$	Multiply y by 4 to get $y = 4x^2$
-3	9	36
-2	4	16
-1	1	4
0	0	0
1	1	4
2	4	16
3	9	36



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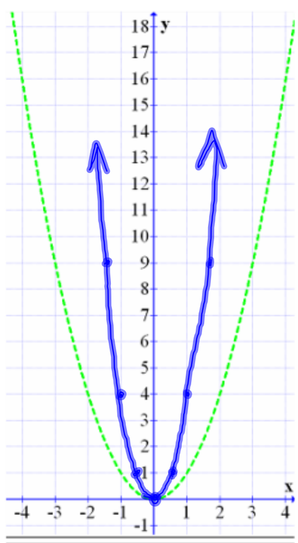
Horizontal compression by a factor of 2: $y = (2x)^2$ $\quad y = f(2x)$

Table of values

divide x by 2 $y = (2x)^2$	x	$y = x^2$
-1.5	-3	9
-1	-2	4
-0.5	-1	1
0	0	0
0.5	1	1
1	2	4
1.5	3	9



Simplify the equation $y = (2x)^2$, what do you notice?

$y = (2x)^2$ ← h.comp. by 2

$= 4x^2$ ← v. stretch by 4

Same!

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Is a reflection along the y axis significant for a parabola? no!

$f(x) = x^2$

$y = f(-x)$

$= (-x)^2$

$= x^2$ ← h. reflection has <u>no effect</u>

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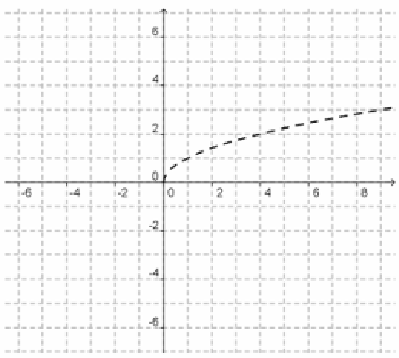
What about these two transformations and the two new functions we have been studying?

For the square root function, compare the two types of scaling.

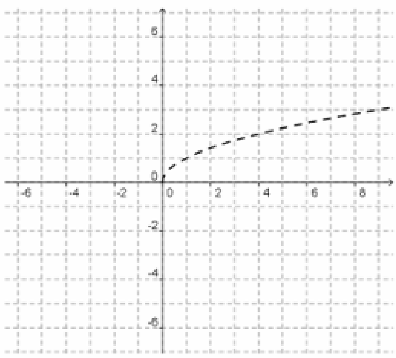
Apply a vertical stretch by a factor of 2.
(Use a table of values with the key points, if you want to!)



Function notation:

Equation:

Apply a horizontal compression by a factor of 4.
(Use a table of values with the key points, if you want to!)



Function notation:

Equation:

Simplified equation:

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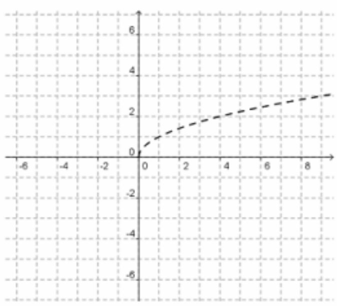
State two ways you could describe the transformation that $y = \sqrt{x}$ has undergone to obtain $y = \sqrt{9x}$.

So why do we bother with horizontal scaling?

Consider the transformation that $y = \sqrt{x}$ has undergone to obtain $y = \sqrt{5x}$, can it be stated two different ways?

For the square root function, compare the two reflections.

Apply a reflection along the x – axis.
(Use a table of values with the key points, if you want to!)



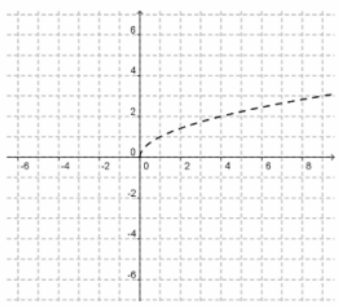
Function notation:

Equation:

Domain:

Range:

Apply a reflection along the y – axis.
(Use a table of values with the key points, if you want to!)



Function notation:

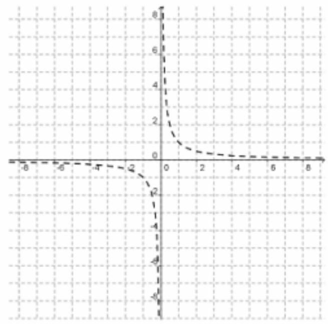
Equation:

Domain:

Range:

Now for the reciprocal function, compare the two types of scaling.

Apply a vertical stretch by a factor of 2.
(Use a table of values with the key points, if you want to!)

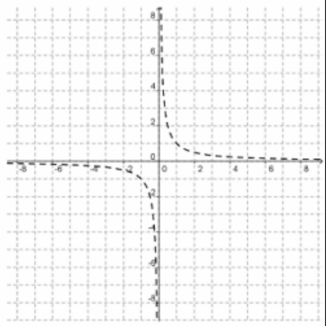


Function notation:

Equation:

What do you notice?

Apply a horizontal stretch by a factor of 2.
(Use a table of values with the key points, if you want to!)



Function notation:

Equation:

Simplified equation:

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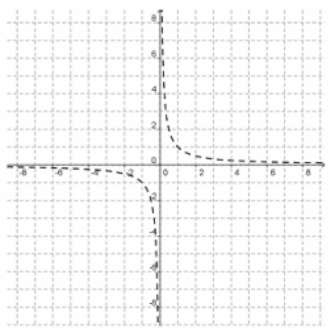
For the reciprocal function, compare the two reflections.

Apply a reflection along the x – axis.
(Use a table of values with the key points, if you want to!)



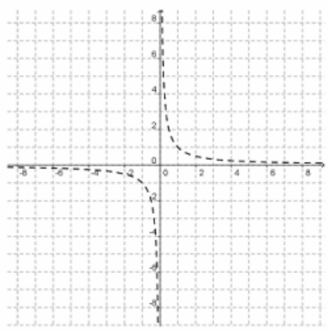
Function notation:

Equation:

Domain:

Range:

Apply a reflection along the y – axis.
(Use a table of values with the key points, if you want to!)



Function notation:

Equation:

Domain:

Range:

What do you notice?

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Homework:

p.240 # 1ef, 2cd, 4dfgh, 5cef, 9cd

Note: Functions must be in the form (see 9d)

$$y = af[k(x-p)]+q$$

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1(e) $y = f\left(\frac{1}{2}x\right) - 6$

$k = \frac{1}{2} \rightarrow$ h. scaling of $\frac{1}{k} = \frac{1}{\frac{1}{2}}$

$= 2$

h. stretch by 2

$q = -6 \rightarrow$ shift down 6

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eg. $y = f(2x) - 6$

$k = 2 \rightarrow$ h. scaling $\frac{1}{k} = \frac{1}{2}$ ✓

h. compression by 2 better

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4(d) $y = f(2x) + 3$

① h. scaling of $\frac{1}{2}$

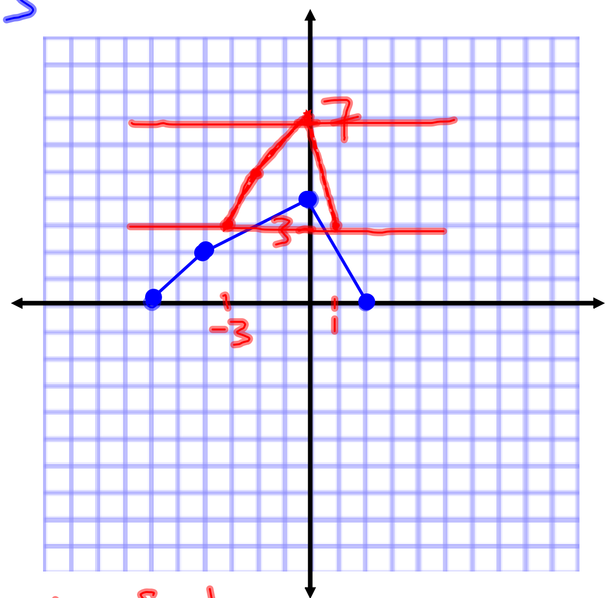
$k=2 \rightarrow$ scale $\frac{1}{k} = \frac{1}{2}$

h. compression by 2

$(x, y) \rightarrow (\frac{x}{2}, y)$

② v. shift by 3

$(\frac{x}{2}, y) \rightarrow (\frac{x}{2}, y+3)$



$D = \{x \mid x \in \mathbb{R}, x \geq -3\ x \leq 1\}$

$R = \{y \mid y \in \mathbb{R}, y \geq 3, y \leq 7\}$

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$y = af[k(x-p)]+q$

$(x,y) \to \left(\frac{x}{k}+p,\ ay+q\right)$

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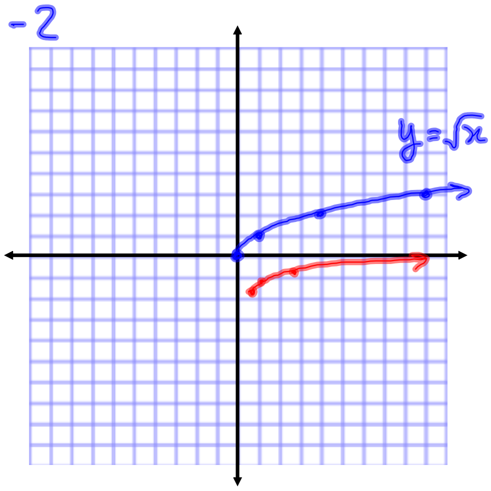
9(c) $y = \frac{1}{2}f(2(x-1)) - 2$

① v. scaling by $\frac{1}{2}$
   v. comp by 2
② h. scaling by $\frac{1}{2}$
   h. comp by 2
③ shift right by 1
④ shift down by 2



$(x,y) \to \left(x, \frac{y}{2}\right) \to \left(\frac{1}{2}x, \frac{1}{2}y\right) \to \left(\frac{1}{2}x+1, \frac{1}{2}y\right)$
$\to \left(\frac{1}{2}x+1, \frac{1}{2}y-2\right)$

$(0,0) \xrightarrow{\frac{1}{2}y} (0,0) \xrightarrow{\frac{1}{2}x} (0,0) \xrightarrow{x+1} (1,0) \xrightarrow{y-2} (1,-2)$
$(1,1) \to (1,0.5) \to (0.5,0.5) \to (1.5,0.5) \to (1.5,-1.5)$
$(4,2) \to (4,1) \to (2,1) \to (3,1) \to (3,-1)$

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