

## Determining Transformed Functions from Graphs

Recall: Given  $y = a f[k(x-p)] + q$   
the transformations are:

1.  $a < 0$ , vertical reflection
2. vertical scaling by  $a$  for  $a \neq 1$
3.  $k < 0$ , horizontal reflection
4. horizontal scaling by  $\frac{1}{k}$  for  $k \neq 1$
5. horizontal translation by  $p$
6. vertical translation by  $q$

$$\begin{aligned}y &= x^2 \\y &= f(2x) \\&= (2x)^2 \\&= 4x^2 \\&= 4f(x)\end{aligned}$$

Mar 6-7:00 PM

Ex.1 Determine the transformation shown and express in function notation.

$$y = a(x-p)^2 + q$$

$$y = a(x+4)^2 + 3$$

sub  $(-3, 1)$

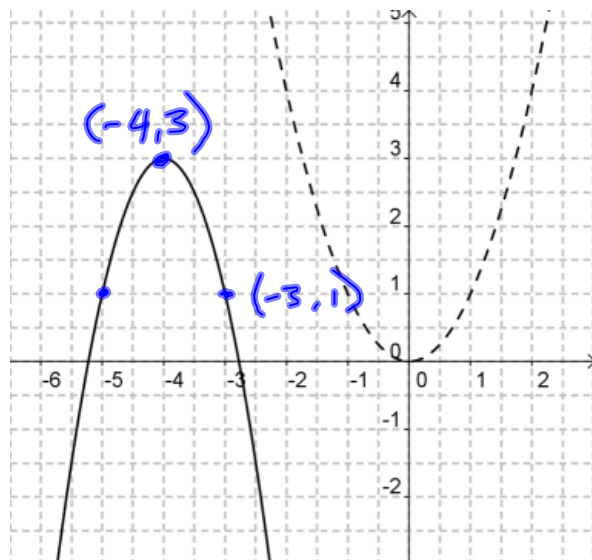
$$1 = a(-3+4)^2 + 3$$

$$1 = a + 3$$

$$\boxed{a = -2}$$

$$y = -2(x+4)^2 + 3 \quad \text{or} \quad y = -2f(x+4) + 3$$

equation function notation



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Ex.2 Determine the transformations shown and express in function notation.

$$p = -2$$

$$q = -3$$

$$f(x) = \sqrt{x}$$

$$y = a f(x-p) + q$$

*↑  
ignoring k  
(for now)*

$$y = a \sqrt{x-p} + q$$

$$y = a \sqrt{x+2} - 3$$

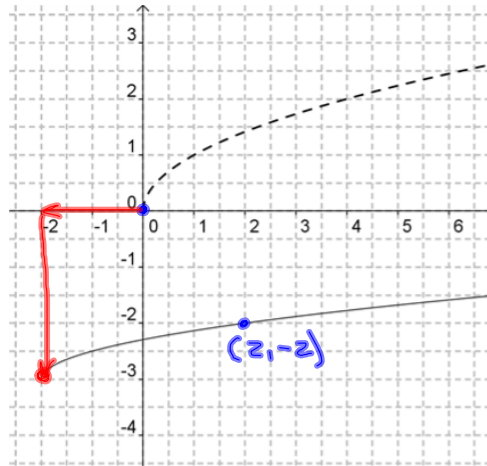
*sub (2,-2)*

$$-2 = a \sqrt{2+2} - 3$$

$$1 = a \sqrt{4}$$

$$1 = 2a$$

$$a = \frac{1}{2}$$



$$y = \frac{1}{2} \sqrt{x+2} - 3$$

*OR*

$$y = \frac{1}{2} f(x+2) - 3$$

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Ex.2 Determine the transformations shown and express in function notation.

$$f(x) = \sqrt{x}$$

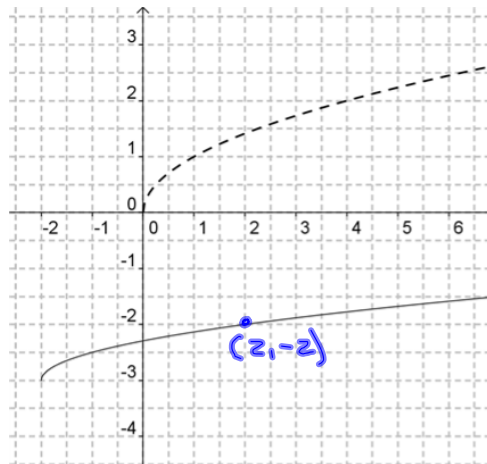
$$f(4x) = \sqrt{4x}$$

$$= \sqrt{4} \sqrt{x}$$

$$= 2 \sqrt{x}$$

$$= 2 f(x)$$

*h. scaling  
equivalent to  
v. scaling*



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Ex.3 Determine the transformations shown and express in function notation.

$$f(x) = \sqrt{x}$$

$$y = a f[k(x-p)] + q$$

$$y = a \sqrt{k(x-p)} + q$$

$$p = 3$$

$$q = 1$$

$$k = -1 \text{ (h. reflect)}$$

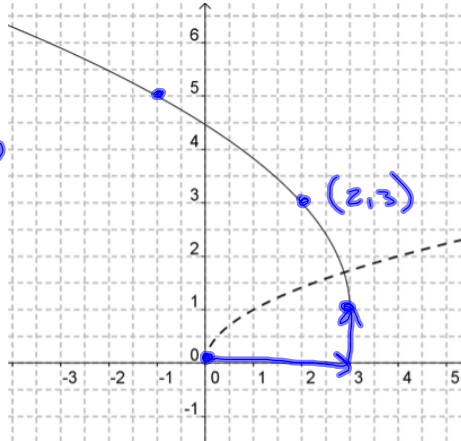
$$y = a \sqrt{-(x-3)} + 1$$

Sub (2,3)

$$3 = a \sqrt{-(2-3)} + 1$$

$$3 = a \sqrt{1} + 1$$

$$2 = a$$



$$y = 2 \sqrt{-(x-3)} + 1$$

$$y = 2 f[-1(x-3)] + 1 \checkmark$$

$$y = 2 f(3-x) + 1 \checkmark$$

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Ex.4 Determine the transformations shown and express in function notation.

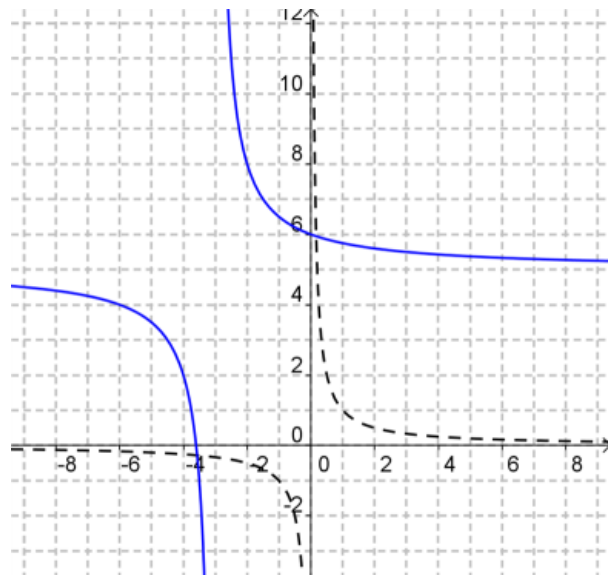
$$f(x) = \frac{1}{x}$$

$$f(2x) = \frac{1}{2x}$$

h. scaling  
equivalent  
to v. scaling

$$= \frac{1}{2} \left( \frac{1}{x} \right)$$

$$\Rightarrow \frac{1}{2} f(x)$$



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Ex.4 Determine the transformations shown and express in function notation.

$$p = -3$$

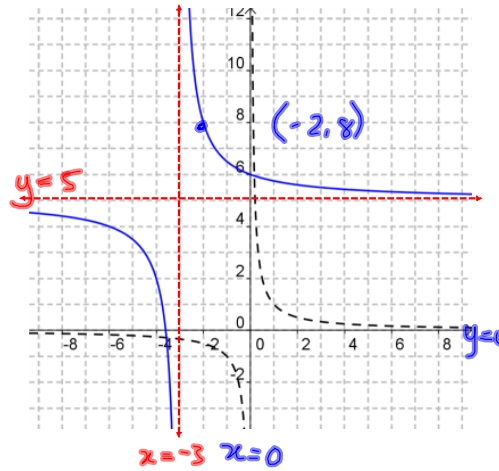
$$q = 5$$

$$f(x) = \frac{1}{x}$$

$$y = a f[k(x-p)] + q$$

$$y = a \left( \frac{1}{k(x-p)} \right) + q$$

ignore



$$y = \frac{a}{x+3} + 5$$

Sub (-2, 8)

$$8 = \frac{a}{-2+3} + 5$$

$$3 = \frac{a}{1}$$

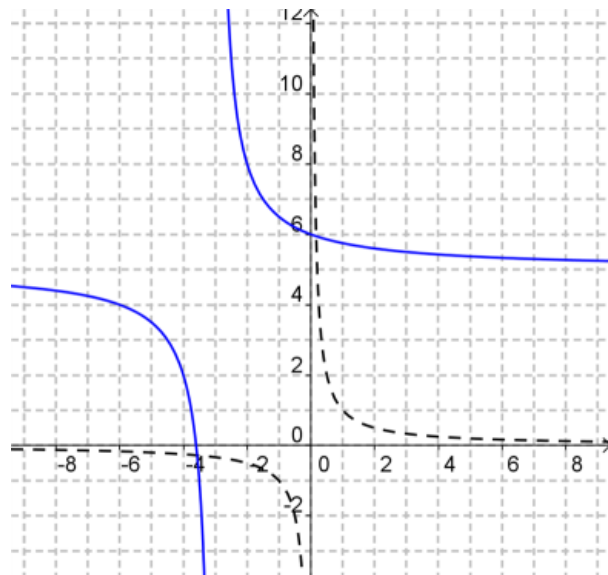
$$a = 3$$

$$y = \frac{3}{x+3} + 5$$

$$y = 3 f(x+3) + 5$$

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Ex.5 Determine the transformations shown and express in function notation.



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Ex.6 Determine the transformations shown and express in function notation.

$$f(x) = ?$$

v. reflect  $a < 0$

h. reflect  $k < 0$

AB: rise 2  
run 1

A'B': rise 6  
run 2

$$\text{rises: } \frac{6}{2} = 3 \therefore a = -3$$

$$\text{runs: } \frac{2}{1} = 2 \therefore k = -\frac{1}{2}$$

$$(x, y) \rightarrow \left( \frac{x}{k} + p, ay + q \right)$$

$$(x, y) \rightarrow (-2x + p, -3y + q)$$

Sub starting point  $\rightarrow$  ending point

$$(4, 0) \rightarrow (-5, -4)$$

$$-2x + p = -5 \text{ when } x = 4$$

$$-2(4) + p = -5$$

$$-8 + p = -5$$

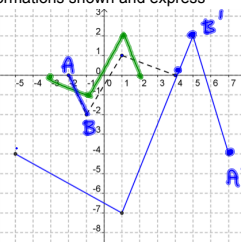
$$p = 3$$

$$-3y + q = -4 \text{ when } y = 0$$

$$-3(0) + q = -4$$

$$q = -4$$

$$y = -3f\left[-\frac{1}{2}(x-3)\right] - 4$$



$$\frac{x}{-\frac{1}{2}} = 2x\left(-\frac{2}{1}\right) = -2x$$

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