Unit 1 - Quadratic Functions & Relations

Finding Max/Min Values by Completing the Square

Jan 31-2:27 PM

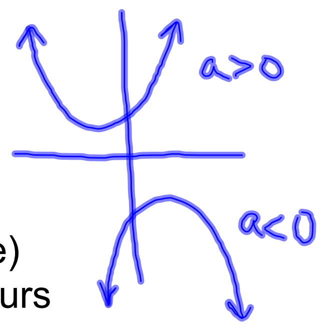
Finding Max/Min Values by Completing the Square

Standard Form		Vertex Form
$y = ax^2 + bx + c$	complete the square $\longrightarrow$	$y = a(x-h)^2 + k$

Vertex is (h, k)
x y

a > 0: opens up (has a minimum)
a < 0: opens down (has a maximum)

k is the <u>optimal value</u> (max or min value)
h is the x-value where the max/min occurs



Feb 6-3:52 PM

Completing the Square Using Fractions:

Ex.1 Complete the square:

(a) $y=3x^2+2x-11$

$$y=3\left(x^2+\frac{2}{3}x\right)-11$$

$$\frac{2}{3}\times\frac{1}{2}=\frac{2}{6}=\frac{1}{3}$$

$$y=3\left(x^2+\frac{2}{3}x+\frac{1}{9}-\frac{1}{9}\right)-11$$

$$\left(\frac{1}{3}\right)^2=\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)=\frac{1}{9}$$

$$y=3\left[\left(x+\frac{1}{3}\right)^2-\frac{1}{9}\right]-\frac{11}{1}\,\begin{matrix}\times 3\\ \times 3\end{matrix}$$

$$3\left(-\frac{1}{9}\right)=-\frac{3}{9}=-\frac{\cancel{3}^{1}}{\cancel{3}\cdot 3}=-\frac{1}{3}$$

$$y=3\left(x+\frac{1}{3}\right)^2-\frac{1}{3}-\frac{33}{3}$$

$$y=3\left(x+\frac{1}{3}\right)^2-\frac{34}{3}$$

$$V\left(-\frac{1}{3},-\frac{34}{3}\right)$$

Feb 6-3:58 PM

(b) $y=5x^2+8x-3$

Feb 7-11:22 AM

Ex.2 Find the optimal value of $y = 5x - 3x^2 + 4$

$y = 5x - 3x^2 + 4$

$y = -3x^2 + 5x + 4$

$\frac{5}{3} \times \frac{1}{2} = \frac{5}{6}$

$\left(\frac{5}{6}\right)^2 = \frac{25}{36}$

$y = -3\left(x^2 - \frac{5}{3}x\right) + 4$

$y = -3\left(x^2 - \frac{5}{3}x + \frac{25}{36} - \frac{25}{36}\right) + 4$

$y = -3\left[\left(x - \frac{5}{6}\right)^2 - \frac{25}{36}\right] + 4$

-11

$= -\frac{33}{3}$

$y = -3\left(x - \frac{5}{6}\right)^2 + \frac{\cancel{3}^{1}(25)}{\cancel{36}_{12}} + 4 \frac{\times 12}{\times 12}$

$\frac{\cancel{3}(25)}{\cancel{3} \cdot 12}$

$y = -3\left(x - \frac{5}{6}\right)^2 + \frac{25}{12} + \frac{48}{12}$

$y = -3\left(x - \frac{5}{6}\right)^2 + \underline{\underline{\frac{73}{12}}}$

$\therefore$ the optimal value is $\frac{73}{12}$

Feb 6-3:58 PM

$y = ax^2 + bx + c$

$y = a\left(x^2 + \frac{b}{a}x\right) + c$

$\frac{b}{a} \times \frac{1}{2}$

$= \frac{b}{2a}$

$y = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right) + c$

$\frac{b^2}{4a^2}$

$y = a\left(\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\right) + c$

Feb 7-11:41 AM

It is permissible, and sometimes preferable, to use terminating decimals (i.e., exact values):

Ex.3 Find the optimal value of $y=-20x^2+180x+4400$

$y=-20x^2+180x+4400$

$\left(-\frac{9}{2}\right)^2$

$=(-4.5)^2$

$=20.25$

$y=-20(x^2-9x)+4400$

$y=-20(x^2-9x+20.25-20.25)+4400$

$y=-20[(x-4.5)^2-20.25]+4400$

$y=-20(x-4.5)^2+405+4400$

$y=-20(x-4.5)^2+\underline{4805}$

max value

opens down

→ max

∴ optimal value is a max of 4805.

Feb 6-3:58 PM

Homework:

p.115 # 1cegik, 3odd, 14, 16

115

Feb 1-7:30 PM

p. 116 # 3(i)

0.5

.025

$y = -2x^2 - 0.8x - 2$

$= -2(x^2 + 0.4x) - 2$

$\times -2$

$= -2(\underbrace{x^2 + 0.4x + 0.04}\ -0.04) - 2$

$= -2[(x+0.2)^2 - 0.04] - 2$

$= -2(x+0.2)^2 + 0.08 - 2$

$= -2(x+0.2)^2 - 1.92$

Feb 8-9:16 AM

3(g) $y = -x^2 - 5x$

$\frac{5}{2}$

$\left(\frac{5}{2}\right)^2 = \frac{25}{4}$

$= -(x^2 + 5x)$

$= -(\underbrace{x^2 + 5x + \tfrac{25}{4}}\ - \tfrac{25}{4})$

$= -\left[\left(x + \tfrac{5}{2}\right)^2 - \tfrac{25}{4}\right]$

$= -\left(x + \tfrac{5}{2}\right)^2 + \tfrac{25}{4}$

$V\left(-\tfrac{5}{2}, \tfrac{25}{4}\right)$

open down → max

max value

$\therefore$ max of $\frac{25}{4}$ when x is $-\frac{5}{2}$

Feb 8-9:22 AM

3(e). $y = -\frac{1}{3}x^2 + 2x + 4$

$$\frac{2}{-\frac{1}{3}} = \frac{2}{1} \times \left(-\frac{3}{1}\right) = -6$$

$y = -\frac{1}{3}(x^2 - 6x) + 4$

$y = -\frac{1}{3}(x^2 - 6x + 9 - 9) + 4$

$y = -\frac{1}{3}\left[(x-3)^2 - 9\right] + 4$

$y = -\frac{1}{3}(x-3)^2 + 3 + 4$

$y = -\frac{1}{3}(x-3)^2 + 7$

$\therefore$ max of 7 when x is 3

Feb 8-9:28 AM