Unit 3 - Rational Expressions

Simplifying Rational Expressions

March 21/2011

A rational number (Q) is the ratio of two integers, a/b, where b is not zero. Dividing anything by zero is <u>undefined</u>.

A rational expression is the ratio of two polynomials. It is also not defined when the denominator is zero (i.e., zeroes of the polynomial in the denominator)

At these undefined values, there is either a "hole" in the graph or an asymptote. This value must be <u>restricted</u>.

Feb 12-9:14 PM

Ex.1 Simplify and state any restrictions

(a) $\dfrac{2x^2 + 10x}{3x + 15}$

$= \dfrac{2x(x+5)}{3(x+5)}$

can divide out (x+5) term,
but x = -5 makes the denominator = 0.

$\dfrac{2x(\cancel{x+5})}{3(\cancel{x+5})}$, $x \neq -5$

$= \dfrac{2x}{3}$, $x \neq -5$

restriction

$2x^2 + 10x$
$= 2x(x+5)$

$3x + 15$
$= 3(x+5)$

Mar 20-11:09 PM

(b) $\dfrac{x^2 - 5x + 6}{x^2 - 4}$

$x^2 - 5x + 6 = (x-2)(x-3)$

$x^2 - 4 = (x-2)(x+2)$

$= \dfrac{(x-2)(x-3)}{(x-2)(x+2)}$

→ cannot be zero
when is it zero?
$(x-2)(x+2) = 0$

$x = 2$ or $x = -2$

$= \dfrac{x-3}{x+2}$; $x \neq \pm 2$

Summary:
1. Factor the numerator and denominator separately.
2. Identify restrictions (where is denominator = 0 ?).
3. Divide out common factors to simplify.

(c) $\dfrac{a^2 - b^2}{b - a}$ → when is $b - a = 0$
$b = a$

$= \dfrac{(a-b)(a+b)}{-(a-b)}$; $a \neq b$

$b - a$
$= -1(-b+a)$
$= -(a-b)$

$= -(a+b)$; $a \neq b$

(d) $\dfrac{x^2 + xy - 2y^2}{y^2 - x^2}$

$= \dfrac{\overset{(1)}{\cancel{(x-y)}}(x+2y)}{(-1)\cancel{(y-x)}(y+x)} \quad ; x \neq \pm y$

$x \neq y \qquad x \neq -y$

$= -\dfrac{x+2y}{x+y} \quad ; \quad x \neq \pm y$

$x^2 + xy - 2y^2$

$= (x-y)(x+2y)$

$x^2 + x - 2$

$= (x-1)(x+2)$

Mar 20-11:12 PM

Two rational expressions are <u>equivalent</u> if they are the same for all possible values in the domain of each expression.

Note: restricted values are <u>not</u> in the domain!

Ex.2 Determine whether expression A is equivalent
to expression B.

A. $\dfrac{2x^2 - 4x - 6}{x+1} \quad ; x \neq -1$

$= \dfrac{2(x^2 - 2x - 3)}{x+1}$

$= \dfrac{2(x-3)\cancel{(x+1)}}{\cancel{x+1}}$

$= 2(x-3)$

$= 2x - 6 \quad ; x \neq -1$

B. $8x^2 - 2x(4x-1) - 6$

$= 8x^2 - 8x^2 + 2x - 6$

$= 2x - 6$

∴ the expressions are equivalent

Mar 20-11:23 PM

3

Homework:

p.40   # 1 - 3(odd) if needed
         # 4 - 6 (odd), 8, 13, 15
         # 16