Multiplying and Dividing Rational Expressions

March 22/2011

recall these operations with fractions:

Simplify each of the following

$\frac{1}{2} \cdot \frac{-3}{5} = \frac{(1)(-3)}{(2)(5)} = -\frac{3}{10}$

$\frac{50}{27} \cdot \frac{-3}{5} = \frac{(\cancel{50}^{10})(-\cancel{3}^{1})}{(\cancel{27}^{9})(\cancel{5}^{1})} = -\frac{10}{9}$

$\frac{12}{27} \div \frac{20}{15} = \frac{\cancel{12}^{4} \cdot \cancel{15}^{3}}{\cancel{27}^{9} \cdot \cancel{20}^{4}} = \frac{1}{3}$

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Multiplying and Dividing rational expressions is very similar to carrying these operations out with fractions, except for the factoring and stating restrictions.

Steps:

1. Factor the numerator and denominator separately.
2. Identify restrictions (where is denominator = 0 ?).
3. Divide out common factors to simplify.

note:

1) In division you have to multiply by the reciprocal of the divisor before step #3.

2) In division, the restrictions are determined from all denominators, before or after taking the reciprocal.

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Ex.1 Simplify and state any restrictions

(a) $\frac{x}{4} \cdot \frac{12^3}{x^2} = \frac{3x^1}{1 \cdot x \cdot x}$

$= \frac{3}{x}$; $x \neq 0$

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(b) $\frac{-5x^3}{3y} \div \frac{y}{25x^2} = \frac{-5x^3}{3y} \cdot \frac{25x^2}{y}$

$y \neq 0$ $x \neq 0$ $y \neq 0$

$= \frac{-125x^5}{3y^2}$

$= -\frac{125x^5}{3y^2}$; $x \neq 0, y \neq 0$

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c) $\dfrac{x+2}{x^2-4x+3} \cdot \dfrac{x-1}{x^2+3x+2}$

$= \dfrac{x+2}{(x-3)(x-1)} \cdot \dfrac{x-1}{(x+1)(x+2)}$

$= \dfrac{1}{(x-3)(x+1)}$; $x \neq -2, -1, 1, 3$

$x^2-4x+3 = (x-3)(x-1)$

$x^2+3x+2 = (x+1)(x+2)$

$x \neq 3, x \neq 1$

$x \neq -1, x \neq -2$

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(d) $\dfrac{2x+4}{x^2-9} \div \dfrac{x^2-4}{x^2-2x-3}$

$= \dfrac{2(x+2)}{(x-3)(x+3)} \cdot \dfrac{(x-3)(x+1)}{(x+2)(x-2)}$

$= \dfrac{2(x+1)}{(x+3)(x-2)}$ $x \neq -3, -2, -1, 2, 3$

or

$x \neq -1, \pm 2, \pm 3$

$2x+4 = 2(x+2)$

$x^2-4 = (x+2)(x-2)$

$x \neq \pm 2$

$x^2-9 = (x+3)(x-3)$

$x \neq -3, x \neq 3$

$x^2-2x-3 = (x-3)(x+1)$

$x \neq -1, 3$

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(e) $\dfrac{x^2-x-20}{x^2-6x} \div \dfrac{x^2+9x+20}{x^2-12x+36}$

$= \dfrac{(x-5)(x+4)}{x(x-6)} \cdot \dfrac{(x-6)^{2\,1}}{(x+5)(x+4)}$

$= \dfrac{(x-5)(x-6)}{x(x+5)}$; $x \neq -5, -4, 0, 6$

$x^2-x-20 = (x-5)(x+4)$

$x^2-6x = x(x-6)$

$x^2+9x+20 = (x+5)(x+4)$

$x^2-12x+36 = (x-6)^2$

$x \neq 0, 6$

$x \neq -5, -4$

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(e) $\left(\dfrac{3a+6}{9a^2} \div \dfrac{a+2}{-3a}\right) \cdot \dfrac{15a}{2}$

* for order of operations, use left to right for operations at the same level (e.g., × or ÷)

$3 \div 4 \cdot 5$

$(3 \div 4) \cdot 5$
$= (0.75)(5)$
$= 3.75$

$3 \div (4 \cdot 5)$
$= 3 \div (20)$
$= 0.15$

$a \neq 0, -2$

$= \dfrac{3(a+2)}{9a^2} \cdot \dfrac{-3a}{a+2} \cdot \dfrac{15a}{2}$

$= -\dfrac{15}{2}$; $a \neq 0, -2$

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Homework:

p.50 # 5ad, 6adgh, 7ae, 8ab, 18

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p. 50 7(e)

$$\frac{2x^2-5x-3}{2x^2-11x+15} \cdot \frac{4x^2-8x-5}{4x^2+4x+1}$$

$$= \frac{(2x+1)(x-3)}{(2x-5)(x-3)} \cdot \frac{(2x-5)(2x+1)}{(2x+1)^2}$$

$= 1, \quad x \neq \frac{5}{2}, 3, -\frac{1}{2}$

$2x^2-5x-3$ S: -5, P: -6, I: -6, 1

$= 2x^2-6x+x-3$

$= 2x(x-3)+1(x-3)$

$= (2x+1)(x-3)$

$2x^2-11x+15$ S: -11, P: 30, I: -5, -6

$2x^2-5x-6x+15$

$= x(2x-5)-3(2x-5)$

$= (2x-5)(x-3)$ *

$4x^2-8x-5$ S: -8, P: -20, I: -10, 2

$= 4x^2-10x+2x-5$

$= 2x(2x-5)+1(2x-5)$

$= (2x-5)(2x+1)$

$4x^2+4x+1$ S: 4, P: 4, I: 2, 2

$= (2x+1)^2$ *

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8(a) $x^2-x-20 = (x-5)(x+4)$

$$\frac{x^2-xy-20y^2}{x^2-8xy+15y^2} \div \frac{x^2+2xy-8y^2}{x^2-xy-6y^2}$$

$$= \frac{(x-5y)(x+4y)}{(x-3y)(x-5y)} \times \frac{(x-3y)(x+2y)}{(x+4y)(x-2y)}$$

$$= \frac{x+2y}{x-2y}, \quad x \neq 3y, 5y, -4y, 2y, -2y$$

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6(h)

$$\frac{2x^2-8}{6x+3} \div \frac{6x-12}{18x+9}$$

$$= \frac{2x^2-8}{6x+3} \cdot \frac{18x+9}{6x-12}$$

$$= \frac{2(x-2)(x+2)}{3(2x+1)} \cdot \frac{9(2x+1)}{6(x-2)}$$

$$= x+2, \quad x \neq -\frac{1}{2}, 2$$

$2x^2-8$

$= 2(x^2-4)$

$= 2(x-2)(x+2)$

$2x+1=0$

$\frac{2x}{2} = -\frac{1}{2}$

$x = -\frac{1}{2}$

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