Problem Solving with Rational Functions

Ex.1. A consultant has issued an environmental report on the cost of cleaning up a property that was previously the site of a chemical factory. Costs can increase dramatically depending on the percent of pollutants that needs to be removed.

Her report ~~gives~~ estimates the cost, C, in dollars, of removing p% of the pollutants from the site as:

$$C(p) = \frac{50000}{100 - p}$$

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a) What is the cost of removal for half of the pollutants? $C(p) = \frac{50000}{100 - p}$

$C(50) = \frac{50000}{100-50}$

$= \frac{50000}{50}$

$= 1000$

∴ the cost to remove 50% of pollutants is $1000

b) What is the cost of removal for 90% of the pollutants?

$C(90) = \frac{50000}{100-90}$

$= \frac{50000}{10}$

$= 5000$

∴ the cost to remove 90% is $5000

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c) Would it be affordable to remove all of the pollutants? $C(p)=\frac{50000}{100-p}$

NO, setting $p=100$ gives an undefined result

→ essentially, an infinite amount of money required.

$p=99.9\%$

$C(99.9)=\frac{50000}{100-99.9}$

$=\frac{50000}{0.1}$

$=500000$

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Ex.2 Fred drove his car a distance of $2x$ km in 3 hours. Later, he drove a distance of $(x + 100)$ km in 2 hours. Use the equation $\text{speed}=\frac{\text{distance}}{\text{time}}$

a) Write an expression for the first speed.

$v_1=\frac{2x}{3}$

b) Write an expression for the second speed.

$v_2=\frac{2x+100}{2}$

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Ex.2 Fred drove his car a distance of $2x$ km in 3 hours. Later, he drove a distance of $(x + 100)$ km in 2 hours. Use the equation $\text{speed} = \frac{\text{distance}}{\text{time}}$

c) Write a simplified expression for the difference between the first speed and the second speed.

$\Delta v = v_2 - v_1$

$= \frac{x+100}{2} - \frac{2x}{3}$ LCD: 6

$= \frac{3(x+100)}{6} - \frac{4x}{6}$

$\Delta v = \frac{-x+300}{6}$

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Ex.2 Fred drove his car a distance of $2x$ km in 3 hours. Later, he drove a distance of $(x + 100)$ km in 2 hours. Use the equation $\text{speed} = \frac{\text{distance}}{\text{time}}$

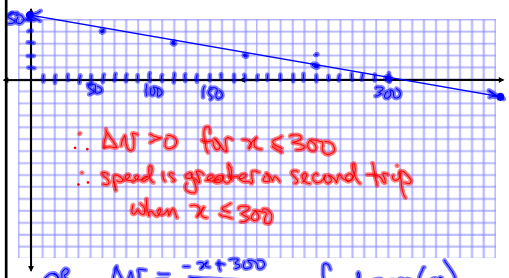
d) Determine the value(s) of x for which the speed was greater for the second trip.

$\Delta v = \frac{-x+300}{6}$

$= -\frac{1}{6}x + 50$

$\Delta v = v_2 - v_1$

Δv will be positive when v_2 is greater.



$\therefore \Delta v > 0$ for $x < 300$

$\therefore$ speed is greater on second trip when $x < 300$

OR $\Delta v = \frac{-x+300}{6}$, find zero(es)

Set $\Delta v = 0$, $0 = \frac{-x+300}{6}$ [x6 both sides]

$0 = -x + 300$

$x = 300$

try value < 300

$x = 200$

$\Delta v = \frac{-200+300}{6}$

$= \frac{100}{6}$

> 0

$x = 400$

$\Delta v = \frac{-400+300}{6}$

$= \frac{-100}{6}$

< 0

$\therefore \Delta v > 0$, v_2 greatest, when $x < 300$

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Ex.2 Fred drove his car a distance of ~~$2x$~~ 3km km in ~~3~~ 2x hours. Later, he drove a distance of ~~$(x+100)$~~ 2km km in ~~2~~ x+100 hours. Use the equation $\text{speed} = \frac{\text{distance}}{\text{time}}$

a) Write an expression for the first speed.

$$v_1 = \frac{3}{2x}$$

b) Write an expression for the second speed.

$$v_2 = \frac{2}{x+100}$$

$$\Delta v = v_2 - v_1$$

$$= \frac{2}{x+100} - \frac{3}{2x}$$

$$= \frac{(2)(2x)}{(x+100)(2x)} - \frac{3(x+100)}{2x(x+100)}$$

$$= \frac{4x - 3x - 300}{2x(x+100)}$$

$$= \frac{x-300}{2x(x+100)}$$

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Assigned Work:

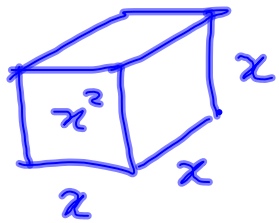
- problem on worksheet
- problems from text (also on WS)

p. 42 #11

$$\frac{V}{SA} = \frac{x^3}{6x^2}$$

$$= \frac{x \cdot x \cdot x}{6 \cdot x \cdot x}$$

$$= \frac{x}{6} \; ; x \neq 0$$



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p.51 #10.

$\ell = \frac{x^2-9}{x}$

$w = \frac{x^3-4x}{x^2+5x+6}$

$x^3-4x = x(x^2-4) = x(x-2)(x+2)$

$A = \ell \cdot w$

$= \frac{(x-3)(x+3)}{x} \cdot \frac{x(x-2)(x+2)}{(x+2)(x+3)}$

$= \frac{(x-3)(x-2)}{1}$

$= (x-3)(x-2)$; $x \neq 0, -2, -3$ holes

$= x^2-5x+6$

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WS

$V(t) = 5000 - \frac{2000t^2}{(t+2)^2}$

C CP P

(a) 4777, 3875, 3530, 3065

5000 ← local maximum

guess no local min.

t

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