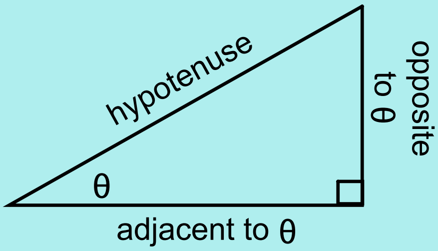
Recall:

For any angle of interest (θ), there are three (3) primary trigonometric ratios.

$$\text{sine of } \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

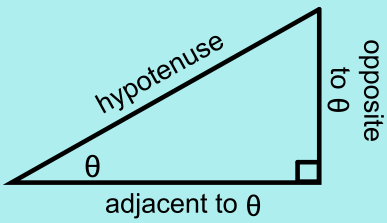
$$\text{cosine of } \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{tangent of } \theta = \frac{\text{opposite}}{\text{adjacent}}$$



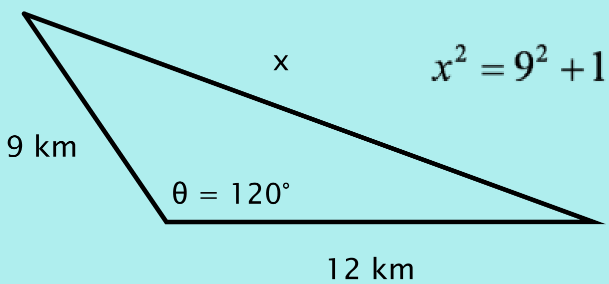
S o h C a h T o a

Our reference triangle (as shown) is generally represented as an acute triangle (i.e., all angles $\leq 90°$).



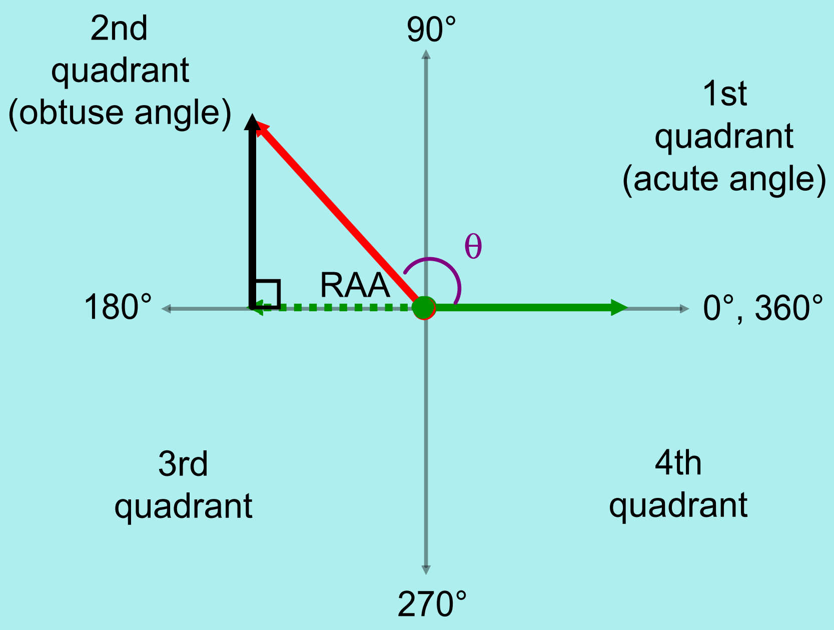
But...

using the cosine law, we have solved triangles such as the one shown below... how?



$$x^2 = 9^2 + 12^2 - 2(9)(12)\cos(120°)$$

To work with angles greater than 90°, we form a right-triangle using the <u>terminal arm</u> and the <u>related acute angle</u>.

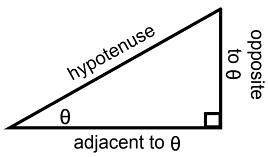


Apr 19-9:19 PM

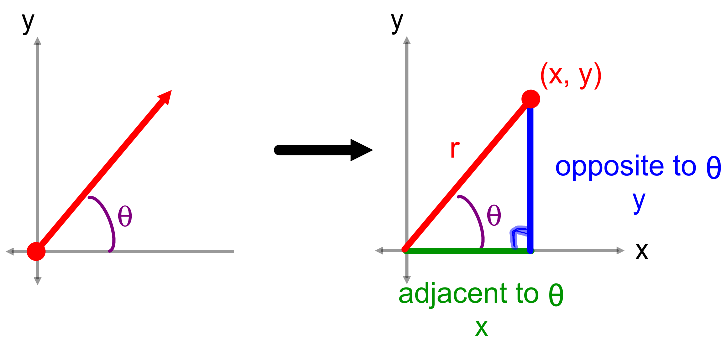
Trigonometry of Any Angle

Apr. 26/2011

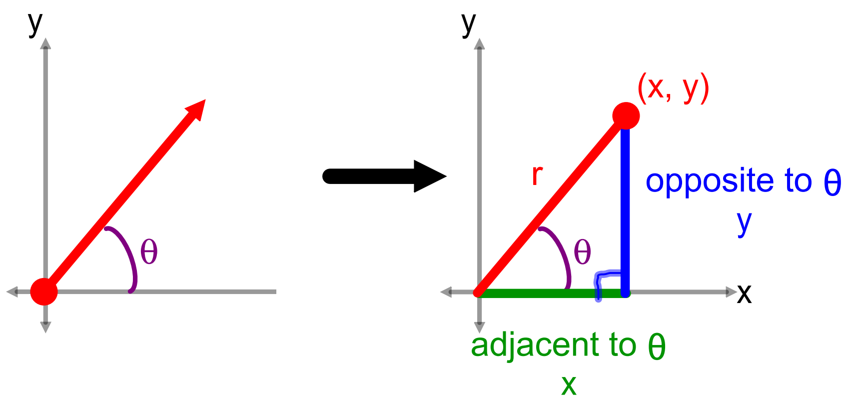
All trigonometric ratios are defined in terms of the sides of an acute right-triangle.



We can redefine the trig ratios for angles in standard position by drawing a right-triangle using the terminal arm.



Apr 19-9:13 PM



where: $r^2 = x^2 + y^2$

$\sin\theta = \dfrac{y}{r}$ $\cos\theta = \dfrac{x}{r}$ $\tan\theta = \dfrac{y}{x}$

This definition refers only to coordinates (x, y) and the distance (r) from the origin to the point, and should be valid for any angle.

Apr 25-10:21 PM

Ex.1 The point (-4, 3) is on the terminal arm of an angle θ in standard position. Find sine and cosine for θ.

$\sin\theta = \frac{y}{r}$

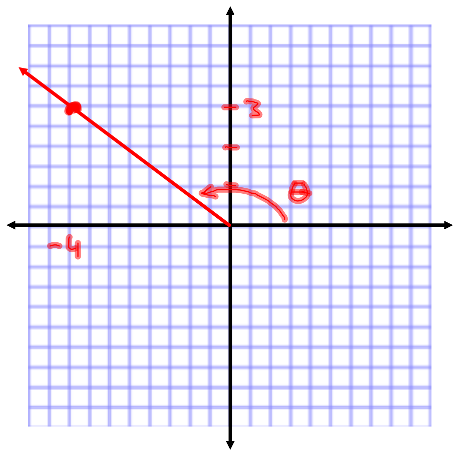
$\cos\theta = \frac{x}{r}$

$r^2 = x^2 + y^2$

$r = \sqrt{3^2 + (-4)^2}$

$r = \sqrt{25}$

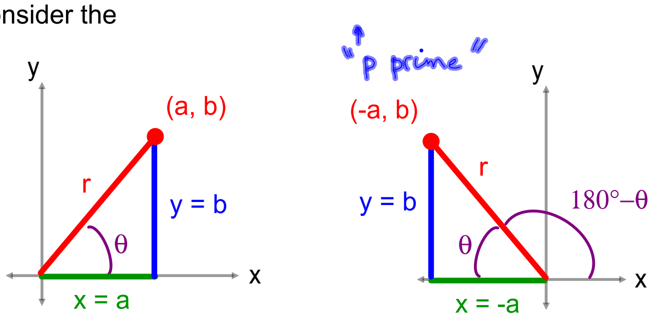
$r = 5$



$\sin\theta = \frac{3}{5}$ $\cos\theta = \frac{-4}{5}$

Apr 25-10:48 PM

Focusing on),
consider the



$\sin\theta = \frac{b}{r}$ $\qquad$ $\sin(180° - \theta) = \frac{b}{r}$

$\therefore$ $\sin\theta = \sin(180° - \theta)$ $\quad$ QI & QII

$\cos\theta = \frac{a}{r}$ $\qquad$ $\cos(180° - \theta) = \frac{-a}{r}$

$\therefore$ $\cos\theta = -\cos(180° - \theta)$

OR $\quad$ in QI & QII

$-\cos\theta = \cos(180° - \theta)$

Apr 25-10:27 PM

Ex.2 Express each of the following as the trig ratio of an acute angle, then confirm your answer.

(a) sin(125°)

$\sin\theta = \sin(180° - \theta)$

$\sin 125° = \sin(180° - 125°)$

$= \sin(55°)$

$LS = \sin 125°$

$= 0.8191$ ✓

$RS = \sin 55°$

$= 0.8191$

(b) cos(160°)

$\cos\theta = -\cos(180° - \theta)$

$\cos 160° = -\cos(180° - 160°)$

$= -\cos(20°)$

$LS = \cos 160°$

$= -0.9396$ ✓

$RS = -\cos 20°$

$= -0.9396$

Apr 25-11:14 PM

Ex.3 Find θ, if $0 \leq \theta \leq 180°$.

(a) $\sin\theta = 0.25$

$\theta = \sin^{-1}(0.25)$

$\theta \doteq 14.5°$

but

$\sin\theta = \sin(180° - \theta)$

$\sin(14.5°) = \sin(180° - 14.5°)$

$= \sin(165.5°)$

$\therefore \theta = 14.5°$ or $\theta = 165.5°$

(b) $\cos\theta = -0.87$

$\theta = \cos^{-1}(-0.87)$

$\theta \doteq 150.5°$

only one answer for cosine when $0 \leq \theta \leq 180°$

Apr 25-11:14 PM

Assigned Work:

p.281# 1
2odd (express in terms of acute angle first)
3odd, 5, 6, 9, 12*

Apr 21-12:17 AM