Recall:

For any angle θ in standard position, where the terminal arm passes through the point (x, y):

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x} \qquad \text{where: } r^2 = x^2 + y^2$$

For angles in Q1 and Q2,

$$\sin\theta = \sin(180° - \theta)$$

- ambiguous
- two angles give the same value for sine

$$\cos\theta = -\cos(180° - \theta)$$

or

$$-\cos\theta = \cos(180° - \theta)$$

- unambiguous
- all angles give unique values for cosine

Apr 25-9:54 PM

Sine Law & Ambiguous Case

Apr. 27/2011

In the x-y plane, trig ratios are expressed in terms of x, y, and r.

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \text{where: } r^2 = x^2 + y^2$$

Since sinθ is positive for both acute (0° to 90°) and obtuse (90° to 180°), there are two angles that yield the same answer for sinθ.

For example,

sin30° = 0.5
sin150° = 0.5

Given sinθ = 0.5, how can we choose between 30° and 150°?

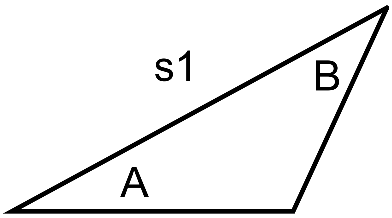
Apr 19-9:13 PM

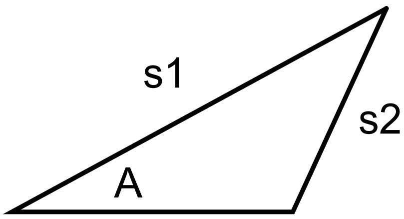
Recall: The Sine Law

The sine law is generally used when we have an oblique (non-right) triangle and:

(a) two angles and the enclosed side (ASA)

(b) two sides and a non-enclosed angle (SSA)





Apr 26-9:36 PM

When we solve for <u>any angle</u> using the sine law, we must consider two possible angles, one acute and one obtuse.

Common sense will often allow us to determine which answer is appropriate. For example:

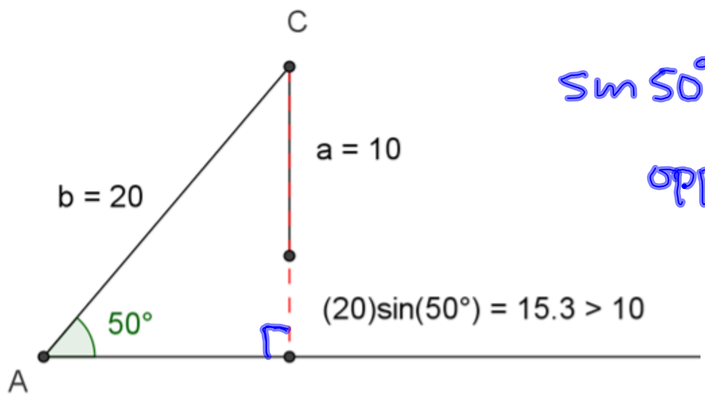
- interior angles must add to 180°
- longer sides correspond to larger angles

With SSA, it is possible to encounter three situations:
(a) no solution - a triangle cannot be formed from the data
(b) one solution - a single triangle is possible
(c) two solutions - two <u>valid</u> triangles can be formed

Apr 26-9:45 PM

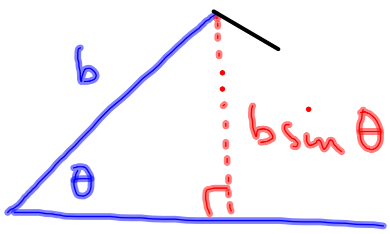
For acute triangles (A < 90°):

Case 1: $a < b \sin A$, too short, no triangle



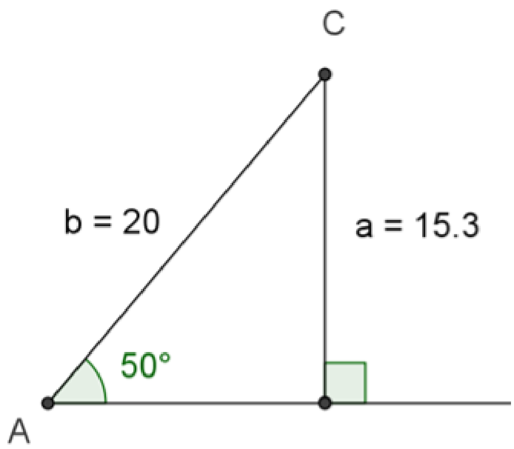
$$\sin 50° = \frac{opp}{20}$$

$$opp = 20 \sin 50°$$



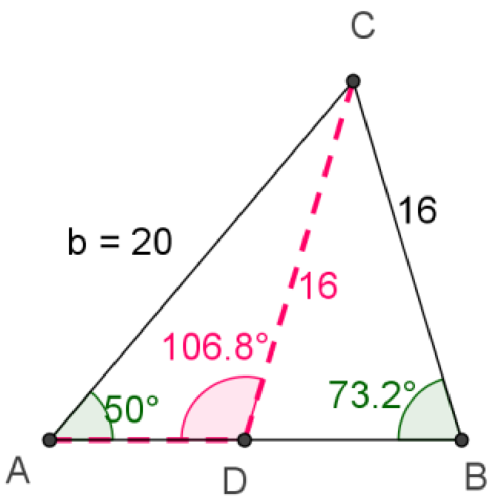
For acute triangles (A < 90°):

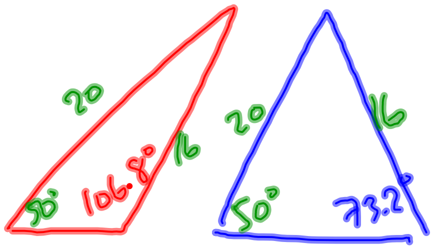
Case 2: $a = b \sin A$, right angle, one solution



For acute triangles (A < 90°):

Case 3: a > b sin A and a < b, two valid solutions

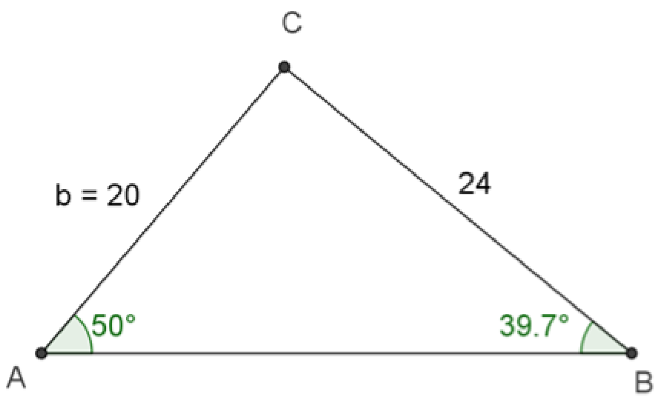




Apr 26-9:54 PM

For acute triangles (A < 90°):

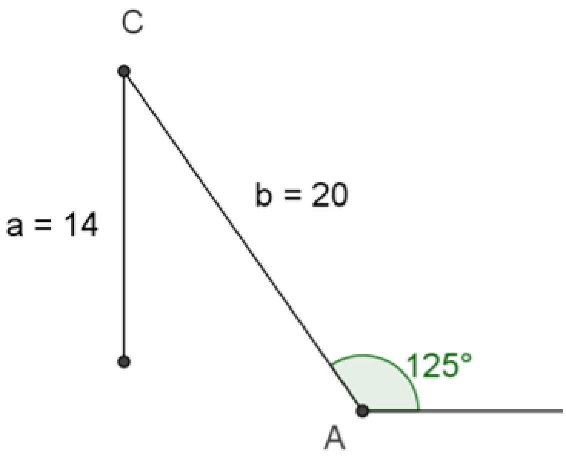
Case 4: a $\geq$ b, one solution



Apr 26-9:54 PM

For obtuse triangles (A > 90°):

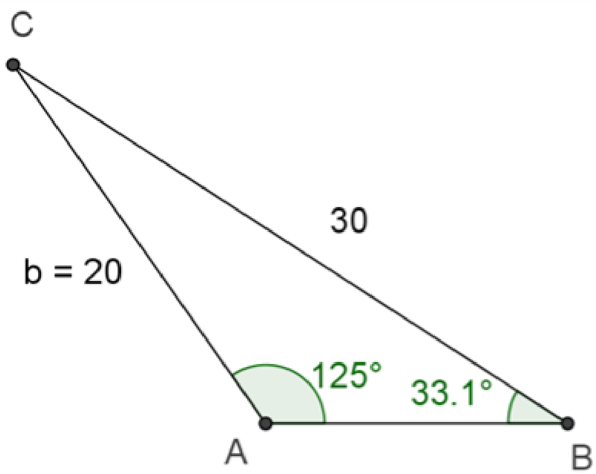
Case 1: a $\leq$ b, too short, no triangle



Apr 26-9:54 PM

For obtuse triangles (A > 90°):

Case 2: a > b, one solution



Apr 26-9:54 PM

Summary:

For acute triangles ($A < 90°$):

Case 1: $a < b \sin A$, too short, no solution
Case 2: $a = b \sin A$, right angle, one solution
Case 3: $a > b \sin A$ and $a < b$, two valid solutions
Case 4: $a \geq b$, one solution

For obtuse triangles ($A > 90°$):

Case 1: $a \leq b$, too short, no solution
Case 2: $a > b$, one solution

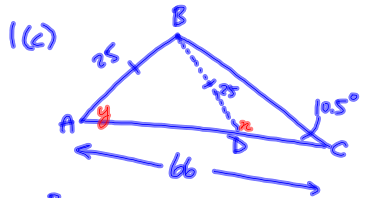
Apr 28-9:46 AM

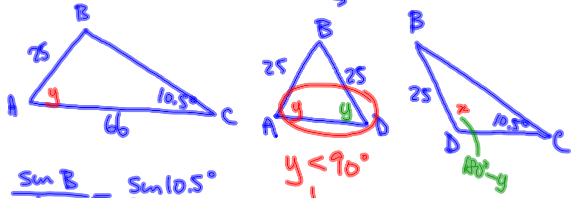
Assigned Work:

Review Ex.1 & 2 from p.303
p.308 # 1ace, 2ace, 3ceg, 4, 8, 13 , 15

Apr 21-12:17 AM

1(c)





$y < 90°$

$$\frac{\sin B}{66} = \frac{\sin 10.5°}{25}$$

$$\sin B = \frac{66 \sin 10.5°}{25}$$

$$B = \sin^{-1}\left(\frac{66 \sin 10.5°}{25}\right)$$

$B \doteq 28.757°$ ✗ or $B \doteq 180° - 28.757°$

$\doteq 151.243°$ ✓

$\Downarrow$ $\Downarrow$

$y = 180° - 28.757° - 10.5°$ $\quad$ $y = 180° - 151.243° - 10.5°$

$= 140.743°$ not possible $\quad$ $= 18.257°$ ✓

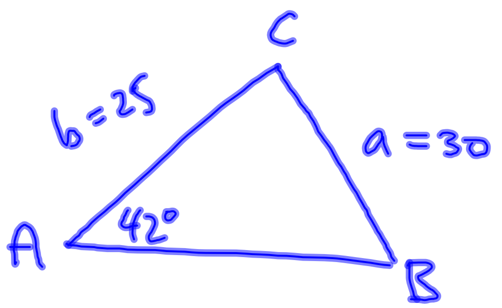
$\Downarrow$ $\Downarrow$

$x = 180° - 140.743°$ $\quad$ $x = 180° - 18.257°$

$x = 39.257°$ $\quad$ $x = 161.743°$ ✓

Apr 28-9:20 AM

2(a) ΔABC $\quad$ $\angle A = 42°$ $\quad$ $a = 30$ $\quad$ $b = 25$



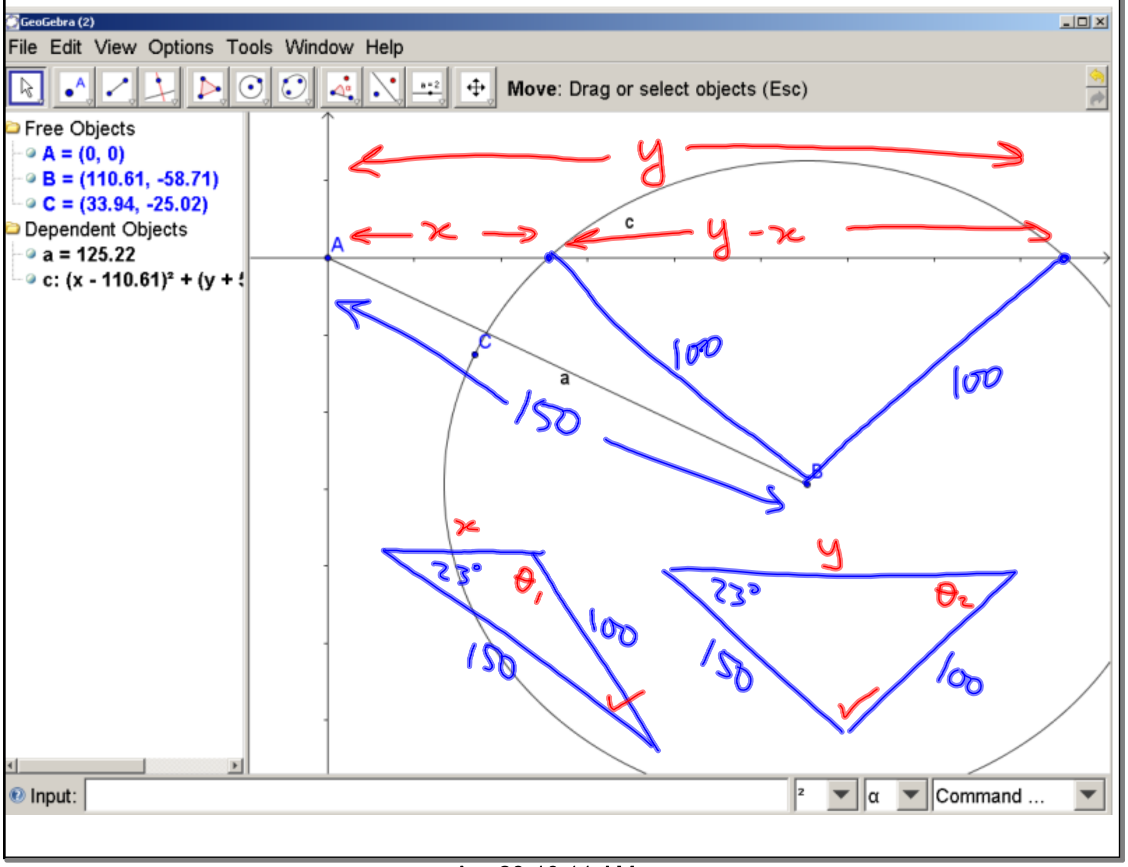
$a \geq b$

one solution

$$\frac{\sin B}{25} = \frac{\sin 42°}{30}$$

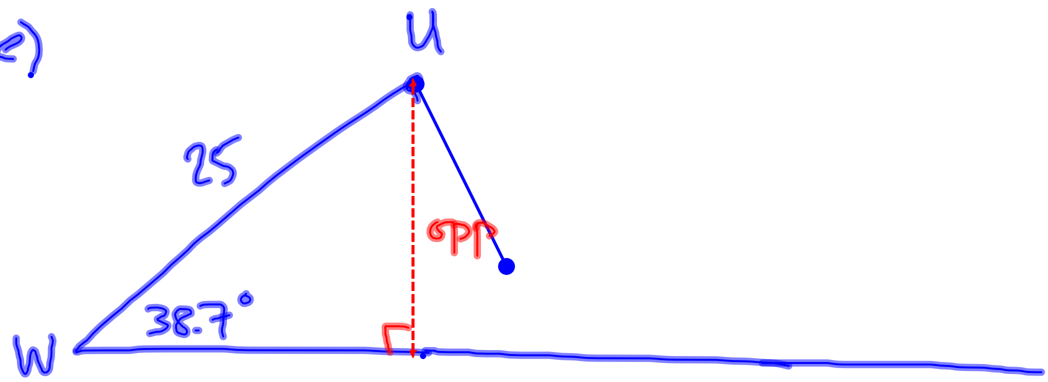
Apr 28-9:55 AM

#8



Apr 28-10:11 AM

2(e)

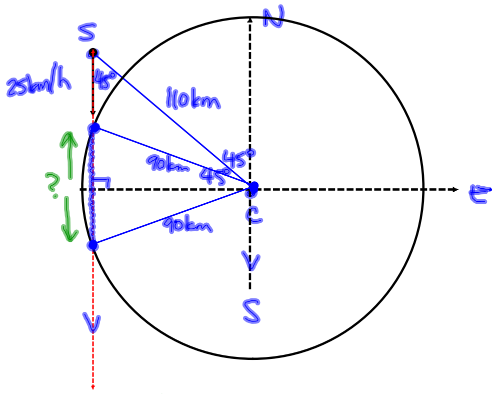


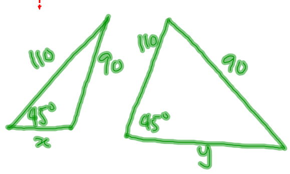
$$\frac{opp}{25} = \sin 38.7^\circ$$

$$opp = 25 \sin 38.7^\circ$$

$$\doteq 15.63$$

Apr 29-10:27 AM





want $y - x$
→ distance where ship is covered by radar.

want time

$d = vt$

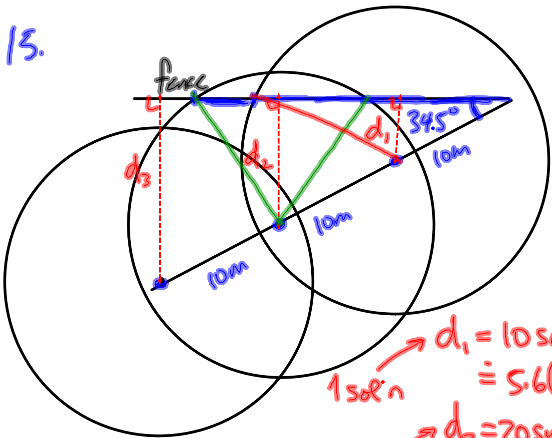
$t = \frac{d}{v}$

$= \frac{y-x}{25}$ ← km, km/h

$\doteq \underline{3.6}$ hours

Apr 29-10:32 AM

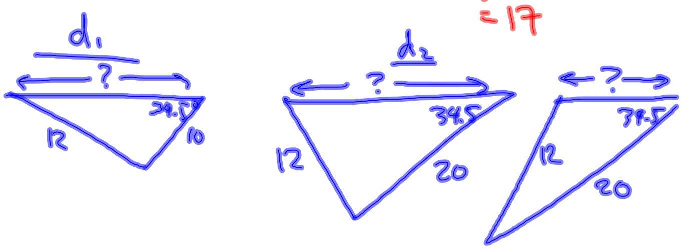
15.



1 sol'n → $d_1 = 10 \sin 34.5°$
$\doteq 5.66$

2 sol'n → $d_2 = 20 \sin 34.5°$
$\doteq 11.32$

0 sol'n → $d_3 \doteq 30 \sin 34.5°$
$\doteq 17$



Apr 29-10:43 AM