Trigonometric Identities

Apr. 29/2011

An <u>identity</u> is an equation which is always true for all values of the variable.

e.g.,
$$(x+1)^2 = x^2 + 2x + 1$$

In the x-y plane, trig ratios are expressed in terms of x, y, and r.

$$\sin \theta = \frac{y}{r}$$
 $\cos \theta = \frac{x}{r}$ $\tan \theta = \frac{y}{x}$ where $r^2 = x^2 + y^2$

We will use these definitions to develop some of the fundamental trig identities.

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1. Quotient Identity

Consider $\frac{\sin \theta}{\cos \theta} = \frac{\frac{1}{2}}{\frac{2}{1}}$ (r > 0) $= \frac{y}{r} \div \frac{z}{r}$ $= \frac{y}{r} \times \frac{r}{x}$ $= \frac{y}{x}$ $= \frac{y}{x}$ $= \frac{y}{x}$

Quotient
$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

Conventions in Trigonometry:

1. Brackets are required around the entire argument of a multi-term argument.

write sin(x+2), not sinx+2 (looks like sin(x) + 2) $sin 25^{\circ}$ is acceptable, as is cos x, or $tan \theta$.

2. Exponents (other than -1) are written between the function symbol and the argument.

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2. Pythagorean Identity

Consider
$$\sin^2 \theta + \cos^2 \theta = (\sin \theta)^2 + (\cos \theta)^2$$

$$= (\frac{1}{r})^2 + (\frac{1}{r})^2$$

$$= \frac{1}{r^2} + \frac{1}{2^2}$$

$$= \frac{1}{r^2} + \frac{1}{2^2}$$

$$= \frac{1}{r^2}$$

$$= \frac{1}{r^2}$$
or $\sin^2 \theta + \cos^2 \theta = 1$
or $\sin^2 \theta = 1 - \cos^2 \theta$
or $\cos^2 \theta = 1 - \sin^2 \theta$

Tips for working with trig identities:

- 1. Start with the most complicated side and try to make it simpler.
- 2. Express tangent in terms of sine and cosine.
- 3. Look for fundamental identities (quotient, Pythagorean).
- 4. Only work on one side at a time. Only switch sides if you cannot progress any further.

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Ex.1 Prove
$$\tan \theta \cos \theta = \sin \theta$$

$$LS = fan \theta \cos \theta \qquad RS = Sm \theta$$

$$= \left(\frac{Sm \theta}{cos \theta}\right) \cdot cos \theta$$

$$= Sin \theta$$

$$\therefore LS = RS$$

$$\therefore Identity is true$$

Ex.2 Prove
$$\tan^2 \theta = \sin^2 \theta \cos^{-2} \theta$$

$$LS = \tan^2 \theta$$

$$RS = \sin^2 \theta \cos^{-2} \theta$$

$$x = (\sin \theta)^2$$

$$x = (\tan \theta)$$

$$x = ($$

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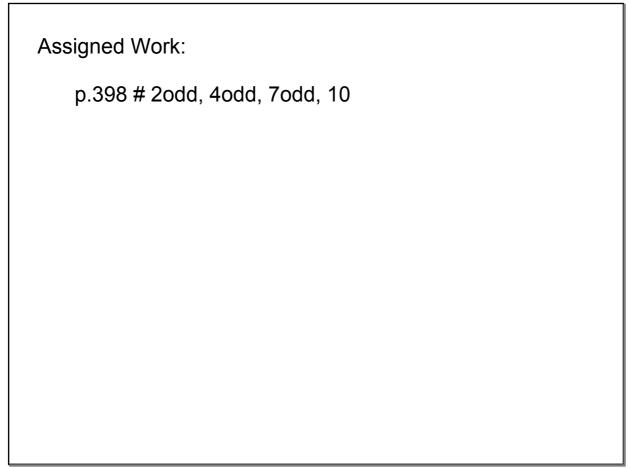
Ex.3 Prove
$$\frac{\sin^2 \theta}{1 - \cos \theta} = 1 + \cos \theta$$

$$(S = \frac{\sin^2 \theta}{1 - \cos \theta})$$

$$= \frac{(-\cos^2 \theta)}{1 - \cos \theta}$$

$$= \frac{(-\cos^2 \theta)}{1 - \cos^2 \theta$$

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