Recall:

In the x-y plane, trig ratios are expressed in terms of x, y, and r.

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$ where: $r^2 = x^2 + y^2$

An identity is an equation which is always true for all values of the variable.

Quotient Identity: $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Pythagorean Identity: $\sin^2\theta + \cos^2\theta = 1$

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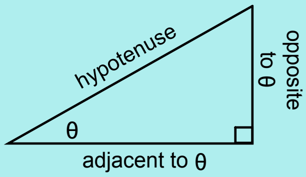
Recall:

For any angle of interest (θ), there are three (3) primary trigonometric ratios.

sine of $\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

cosine of $\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

tangent of $\theta = \frac{\text{opposite}}{\text{adjacent}}$



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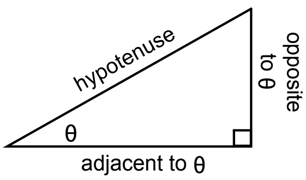
Trigonometric Identities (continued) May 2/2011

The secondary trig ratios provide the reciprocal identities, and are defined as follows:

cosecant of $\theta = \frac{\text{hypotenuse}}{\text{opposite}}$
csc θ

secant of $\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$
sec θ

cotangent of $\theta = \frac{\text{adjacent}}{\text{opposite}}$
cot θ



They are the reciprocals of the primary trig ratios:

$\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

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Using the reciprocal identities, consider dividing $\sin^2\theta + \cos^2\theta = 1$ by $\cos^2\theta$.

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\boxed{\tan^2\theta + 1 = \sec^2\theta}$$

$$\tan^2\theta = \sec^2\theta - 1$$

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Using the reciprocal identities, consider dividing $\sin^2\theta+\cos^2\theta=1$ by $\sin^2\theta$.

$$\frac{\sin^2\theta}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}=\frac{1}{\sin^2\theta}$$

$$\boxed{1+\cot^2\theta=\csc^2\theta}$$

$$\cot^2\theta=\csc^2\theta-1$$

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Summary:

Quotient Identity:

$$\tan\theta=\frac{\sin\theta}{\cos\theta}$$

Pythagorean Identity:

$$\sin^2\theta+\cos^2\theta=1$$

Reciprocal Identities:

$$\csc\theta=\frac{1}{\sin\theta}\qquad \sec\theta=\frac{1}{\cos\theta}\qquad \cot\theta=\frac{1}{\tan\theta}$$

Other Useful Identities:

$$\tan^2\theta+1=\sec^2\theta\qquad \cot^2\theta+1=\csc^2\theta$$

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Ex.1 Prove $\csc\theta=\frac{\cot\theta}{\cos\theta}$ (#1 from WS 3.3)

$$RS=\frac{\frac{1}{\tan\theta}}{\cos\theta}$$

$$=\frac{1}{\tan\theta}\div\cos\theta$$

$$=\frac{1}{\frac{\sin\theta}{\cos\theta}}\cdot\frac{1}{\cos\theta}$$

$$=\frac{\cancel{\cos\theta}}{\sin\theta}\cdot\frac{1}{\cancel{\cos\theta}}$$

$$=\frac{1}{\sin\theta}$$

$$=\csc\theta$$

$\because LS=RS$

$\therefore$ identity is true

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Ex.2 Prove $\csc^4 x-\cot^4 x=\csc^2 x+\cot^2 x$
(#5 from WS 3.3)

$LS=\csc^4 x-\cot^4 x$ $\qquad a^2-b^2=(a-b)(a+b)$

$$=(\csc^2 x-\cot^2 x)(\csc^2 x+\cot^2 x)$$

$$=(1+\cot^2 x-\cot^2 x)(\csc^2 x+\cot^2 x)$$

$$=\csc^2 x+\cot^2 x$$

$\because LS=RS$

$\therefore$ identity is true

OR

$$=\left(\frac{1}{\sin^2 x}-\frac{\cos^2 x}{\sin^2 x}\right)(\sim\sim)$$

$$=\left(\frac{1-\cos^2 x}{\sin^2 x}\right)(\sim\sim)$$

$$=\left(\frac{\sin^2 x}{\sin^2 x}\right)(\sim\sim)$$

$$=(\csc^2 x+\cot^2 x)$$

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Assigned Work:

WS 3.3 # 2, 3, 4, 6, 9, 11, 12, 14, 15

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3. $LS = \csc^2 y \tan^2 y - 1$

$= \frac{1}{\cancel{\sin^2 y}} \cdot \frac{\cancel{\sin^2 y}}{\cos^2 y} - 1$

$= \frac{1}{\cos^2 y} - 1$

$= \frac{1 - \cos^2 y}{\cos^2 y}$

$= \frac{\sin^2 y}{\cos^2 y}$

$= \tan^2 y$

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15. $LS = \frac{\sin y + \tan y}{1 + \sec y}$

$= \frac{\sin y + \frac{\sin y}{\cos y}}{1 + \frac{1}{\cos y}}$

$= \frac{\frac{\sin y \cos y + \sin y}{\cos y}}{\frac{\cos y + 1}{\cos y}}$

$= \frac{(\sin y \cos y + \sin y)}{\cancel{(\cos y)}} \times \frac{\cancel{(\cos y)}}{(\cos y + 1)}$

$= \frac{\sin y \cos y + \sin y}{\cos y + 1}$

$= \frac{\sin y \cancel{(\cos y + 1)}}{\cancel{(\cos y + 1)}}$

$xy + x$
$= x(y+1)$

$= \sin y$

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9. $LS = \frac{\cot w}{\cos w} + \frac{\sec w}{\cot w}$

$\cot w = \frac{1}{\tan w}$
$= \frac{1}{\frac{\sin w}{\cos w}}$
$= \frac{\cos w}{\sin w}$

$= \frac{\frac{\cos w}{\sin w}}{\frac{\cos w}{1}} + \frac{\frac{1}{\cos w}}{\frac{\cos w}{\sin w}}$

$= \frac{\cancel{\cos w}}{\sin w} \cdot \frac{1}{\cancel{\cos w}} + \frac{1}{\cos w} \cdot \frac{\sin w}{\cos w}$

$= \frac{1}{\sin w} + \frac{\sin w}{\cos^2 w}$

$= \frac{\cos^2 w + \sin^2 w}{\sin w \cos^2 w}$

$= \frac{1}{\sin w \cos^2 w}$

$= \frac{1}{\sin w} \cdot \frac{1}{\cos^2 w}$

$= \csc w \cdot \sec^2 w$

$= \sec^2 w \csc w$

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4. $LS = \dfrac{\sec\theta}{\cos\theta} - \dfrac{\tan\theta}{\cot\theta}$

$= \dfrac{\frac{1}{\cos\theta}}{\underbrace{\frac{\cos\theta}{}}_{1}} - \dfrac{\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\sin\theta}}$

$= \dfrac{1}{\cos\theta} \cdot \dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta} \cdot \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{1}{\cos^2\theta} - \dfrac{\sin^2\theta}{\cos^2\theta}$

$= \dfrac{1 - \sin^2\theta}{\cos^2\theta}$

$= \dfrac{\cos^2\theta}{\cos^2\theta}$

$= 1$

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