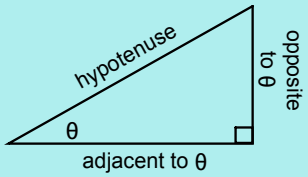
Recall:

For any angle of interest (θ), there are three (3) primary trigonometric ratios.

$$\text{sine of } \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{cosine of } \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

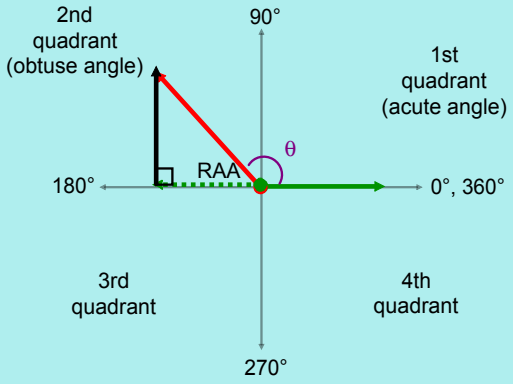
$$\text{tangent of } \theta = \frac{\text{opposite}}{\text{adjacent}}$$



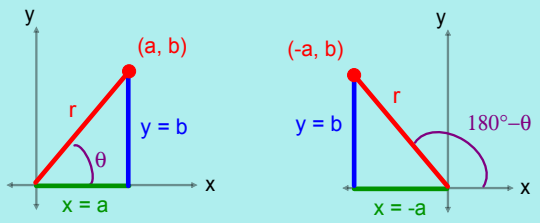
S o h C a h T o a

Apr 25-9:54 PM

To work with angles greater than 90°, we form a right-triangle using the terminal arm and the related acute angle.



Apr 19-9:19 PM



$$\sin\theta = \sin(180° - \theta)$$

$$\cos\theta = -\cos(180° - \theta)$$

or

$$\cos(180° - \theta) = \cos\theta$$

Apr 25-10:27 PM

Trigonometry of Any Angle: The CAST Rule

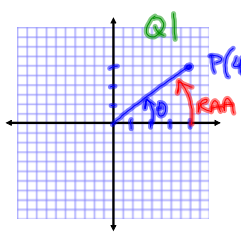
May 3/2011

Any angle in standard position has a related acute angle. An acute right-triangle can always be drawn using this RAA, thus any angle can be associated with the primary trig ratios.

The quadrant will determine the sign of the ratio.

Apr 19-9:13 PM

Ex.1 Consider P(4, 3)

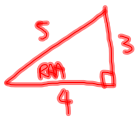


$x = 4 \quad y = 3 \quad r = 5$

$\sin\theta = \frac{y}{r}$ (+) $= \frac{3}{5}$

$\cos\theta = \frac{x}{r}$ (+) $= \frac{4}{5}$

(+) $\tan\theta = \frac{y}{x} = \frac{3}{4}$



$\sin(RAA) = \frac{3}{5}$

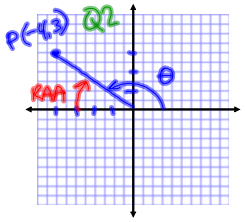
$RAA = \sin^{-1}\left(\frac{3}{5}\right)$

$RAA \doteq 37°$

$\therefore \theta \doteq 37°$

Apr 19-9:13 PM

Ex.2 Consider P(-4, 3)



$x = -4 \quad y = 3 \quad r = 5$

$\sin\theta = \frac{3}{5}$ (+)

$\cos\theta = \frac{-4}{5}$ (−)

$\tan\theta = -\frac{3}{4}$ (−)

$\cos(RAA) = \frac{4}{5}$

$RAA = \cos^{-1}\left(\frac{4}{5}\right)$

$RAA \doteq 37°$

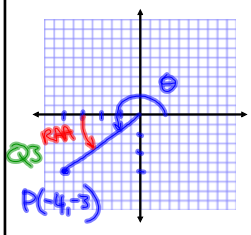
$\therefore \theta = 180° - RAA$

$\doteq 180° - 37°$

$= 143°$

Apr 19-9:13 PM

Ex.3 Consider P(-4, -3)



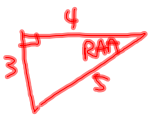
$x = -4 \quad y = -3 \quad r = 5$

$\sin\theta = \frac{-3}{5}$ (−)

$\cos\theta = \frac{-4}{5}$ (−)

$\tan\theta = \frac{-3}{-4}$ (+)

$= \frac{3}{4}$



$RAA \doteq 37°$

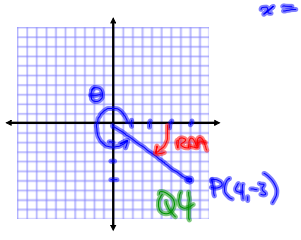
$\theta = 180° + RAA$

$= 180° + 37°$

$= 217°$

Apr 19-9:13 PM

Ex.4 Consider P(4, -3)

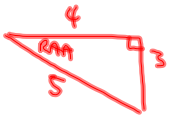


$x = 4 \quad y = -3 \quad r = 5$

$\sin\theta = \frac{-3}{5}$ (−)

$\cos\theta = \frac{4}{5}$ (+)

$\tan\theta = \frac{-3}{4}$ (−)



$RAA \doteq 37°$

$\theta = 360° - RAA$

$= 360° - 37°$

$= 323°$

Apr 19-9:13 PM

The CAST rule allows us to quickly determine the sign of each trig ratio for any quadrant.

Q2	Q1
sin + cos - tan -	sin + cos + tan +
Q3	**Q4**
sin - cos - tan +	sin - cos + tan -

Sin	All
Tan	Cos

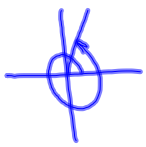
May 3-9:19 AM

Ex.5 Predict the sign of each value (verify with calculator)

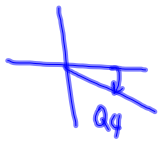
(a) tan 135° (−)

(b) cos 240° (−)

(c) sin 430° = sin 70° (+)



(d) tan(-30°) (−)



May 3-9:22 AM

Ex.6 For $\tan\theta = -\frac{5}{24}$, where $0° \le \theta \le 360°$

(a) where (which quadrant) is θ?

tan negative → Q2 or Q4

(b) determine the primary trig ratios

$\tan\theta = \frac{y}{x} = -\frac{5}{24}$

in Q2

$x = -24$

$y = 5$

$r^2 = x^2 + y^2$

$= (-24)^2 + 5^2$

$= 576 + 25$

$= 601$

$r = \sqrt{601}$

$\sin\theta = \frac{y}{r} = \frac{5}{\sqrt{601}}$

$\cos\theta = \frac{x}{r} = -\frac{24}{\sqrt{601}}$

in Q4

$x = 24$

$y = -5$

$r = \sqrt{601}$

$\sin\theta = \frac{-5}{\sqrt{601}}$

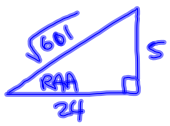
$\cos\theta = \frac{24}{\sqrt{601}}$

May 3-9:24 AM

Ex.6 For $\tan\theta = -\frac{5}{24}$, where $0° \le \theta \le 360°$

(c) determine the value of θ to the nearest degree

(using RAA)



$\tan(RAA) = \frac{5}{24}$

$RAA = \tan^{-1}\left(\frac{5}{24}\right)$

$RAA \doteq 11.8°$

in Q2

$\theta = 180° - RAA$

$\doteq 180° - 11.8°$

$\doteq 168.2°$

in Q4

$\theta = 360° - RAA$

$\doteq 360° - 11.8°$

$\doteq 348.2°$

$\therefore \theta = 168.2°$ (in Q2) or $\theta = 348.2°$ (in Q4)

May 3-9:24 AM

Assigned Work:

TBD

p.281 #3 a-h

Apr 21-12:17 AM