Working With Radicals

Feb 9/2011

Recall: The simplest quadratic relation is $y = x^2$

On rearranging, it is possible to get answers
in the form $x = \pm\sqrt{y}$

With actual values, we might see results such as

$$\sqrt{5} \qquad 3\sqrt{2} \qquad \frac{\sqrt{3}}{2}$$

It is often required to keep answers in this exact form.

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index
understood
to be 2

index of 3

$\sqrt{5}$ radical sign $\sqrt[3]{5}$

radicand

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Multiplying & Dividing Radicals

In general, $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

let $a=4, b=9$

LS $= \sqrt{4}\sqrt{9}$

$= (2)(3)$

$= 6$

RS $= \sqrt{4 \cdot 9}$

$= \sqrt{36}$

$= 6$

and $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ *where* $b \neq 0$

"not equal to"

Ex.1

(a) $\sqrt{27}$

$= \sqrt{3 \cdot 9}$

$= \sqrt{3}\sqrt{9}$

$= 3\sqrt{3}$

(b) $\sqrt{\frac{16}{9}}$ $= \frac{\sqrt{16}}{\sqrt{9}}$

$= \frac{4}{3}$

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Simplifying Radicals

A radical is in its simplest form when:

- the radicand has no perfect square factors (other than 1) $\sqrt{8} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$

- the radicand contains no fractions $\sqrt{\frac{1}{4}} = \frac{1}{2}$

- no radical appears in the denominator $\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

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Ex.2 Simplify

(a) $\sqrt{32}$

$= \sqrt{2 \cdot 16}$

$= 4\sqrt{2}$

(b) $2\sqrt{75}$

$= 2\sqrt{3 \cdot 5 \cdot 5}$

$= 2 \cdot 5\sqrt{3}$

$= 10\sqrt{3}$

(c) $-3\sqrt{8}$

$= -3\sqrt{2 \cdot 2 \cdot 2}$

$= -6\sqrt{2}$

(d) $\frac{1}{2}\sqrt{\frac{72}{25}} = \frac{1}{2} \cdot \frac{\sqrt{8 \cdot 9}}{5}$

$= \frac{3\sqrt{8}}{10}$

$= \frac{3\sqrt{2 \cdot 2 \cdot 2}}{10}$

$= \frac{\cancel{6}^{3}\sqrt{2}}{\cancel{10}_{5}}$

$= \frac{3\sqrt{2}}{5}$

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Homework:

p.106 # (1 - 4)(odd)

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p.106 #1(m)

$$\sqrt{128} = \sqrt{2 \cdot 64}$$

$$= 8\sqrt{2}$$

$$\sqrt{128} = \sqrt{\underbrace{2 \cdot 2}_{2} \cdot \underbrace{2 \cdot 2}_{2} \cdot \underbrace{2 \cdot 2}_{2} \cdot 2}$$

$$= 8\sqrt{2}$$

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p.106 4(a)

$$\frac{\cancelto{2}{10} + \cancelto{3}{15}\sqrt{5}}{\cancelto{1}{5}}$$

$$= \frac{\cancelto{2}{10}}{\cancelto{1}{5}} + \frac{\cancelto{3}{15}\sqrt{5}}{\cancelto{1}{5}}$$

$$= 2 + 3\sqrt{5}$$

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2(c)

$$\frac{\sqrt{60}}{\sqrt{3}} = \sqrt{\frac{\cancel{60}^{20}}{\cancel{3}_1}}$$

$$= \sqrt{20}$$

$$= \sqrt{2 \cdot 2 \cdot 5}$$

$$= 2\sqrt{5}$$

$$\frac{\sqrt{60}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{\sqrt{180}}{3}$$

$$= \frac{\sqrt{2 \cdot 2 \cdot 9 \cdot 5}}{3}$$

$$= \frac{\cancel{6}^{2}\sqrt{5}}{\cancel{3}_1}$$

$$= 2\sqrt{5}$$

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Adding & Subtracting Radicals

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To add or subtract radicals, they must have the same radicand. It is advisable to simplify radicals to ensure all like terms (same radicand) are revealed.

Ex.3 Simplify

(a) $4\sqrt{3} - 2\sqrt{5} + 6\sqrt{3} + 5\sqrt{5}$

$= 10\sqrt{3} + 3\sqrt{5}$

similar

$4x - 2y + 6x + 5y$

$= 10x + 3y$

(b) $2\sqrt{12} - 5\sqrt{27} + 3\sqrt{48}$

$= 2(2\sqrt{3}) - 5(3\sqrt{3}) + 3(4\sqrt{3})$

$= 4\sqrt{3} - 15\sqrt{3} + 12\sqrt{3}$

$= \sqrt{3}$

rough

$\sqrt{12} = \sqrt{3 \cdot 4} = 2\sqrt{3}$

$\sqrt{27} = \sqrt{9 \cdot 3} = 3\sqrt{3}$

$\sqrt{48} = \sqrt{2 \cdot 24}$

$= \sqrt{2 \cdot 2 \cdot 12}$

$= \sqrt{2 \cdot 2 \cdot 2 \cdot 6}$

$= \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}$

$= 4\sqrt{3}$

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Binomial Multiplication of Radicals

Recall: $(a+b)(c+d) = \cancel{ab} + ad + bc + bd$

ac

FOIL

F O I L

Ex.4 Expand & Simplify

$(3\sqrt{5}+2)(2\sqrt{5}-3)$ ⟷ Similar $(3x+2)(2x-3)$

$= 6\sqrt{25} - 9\sqrt{5} + 4\sqrt{5} - 6$ ⟷ $= 6x^2 - 9x + 4x - 6$

5

$= 30 - 5\sqrt{5} - 6$ $\quad = 6x^2 - 5x - 6$

$= 24 - 5\sqrt{5}$

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Rationalizing the Denominator

A radical is not permitted in the denominator. If the denominator is a binomial, multiply by the <u>conjugate</u> of the denominator.

Given $a\sqrt{b} + c\sqrt{d}$, the conjugate would be $a\sqrt{b} - c\sqrt{d}$

Given $a\sqrt{b} - c\sqrt{d}$, the conjugate would be $a\sqrt{b} + c\sqrt{d}$

Ex.5 Find the conjugate of each radical

(a) $\sqrt{5} - \sqrt{2}$

conjugate is

$\sqrt{5} + \sqrt{2}$

(b) $3\sqrt{5} + 2\sqrt{10}$

$3\sqrt{5} - 2\sqrt{10}$

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Ex.6 Rationalize the denominator

$$\frac{4\sqrt{3}-2\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = 1$$

$$= \frac{(4\sqrt{3}-2\sqrt{2})(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$$

$$= \frac{4\sqrt{9}+4\sqrt{6}-2\sqrt{6}-2\sqrt{4}}{\sqrt{9}\underbrace{+\sqrt{6}-\sqrt{6}}_{=0}-\sqrt{4}}$$

$$= \frac{12+2\sqrt{6}-4}{3-2}$$

$$= \frac{8+2\sqrt{6}}{1}$$

$$= 8+2\sqrt{6}$$

Recall

$(x-y)(x+y)$
$= x^2-y^2$

* difference of squares

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Homework:

~~p.106 # (1 - 4)(odd)~~ already assigned
p.139 # (1 - 7)(odd)

6(c) $\frac{2}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$

$$= \frac{2\sqrt{7}}{\sqrt{49}}$$

$$= \frac{2\sqrt{7}}{7}$$

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5(e) $\sqrt{2}(\sqrt{3}+4)$

$= \sqrt{6} + 4\sqrt{2}$

what if?

$\sqrt{9} + 4\sqrt{2}$

$= 3 + 4\sqrt{2}$

4(g) $3\sqrt{48} - 4\sqrt{8} + 4\sqrt{27} - 2\sqrt{72}$

$= 3\sqrt{16\cdot 3} - 4\sqrt{4\cdot 2} + 4\sqrt{9\cdot 3} - 2\sqrt{36\cdot 2}$

×4 ×2 ×3 ×6

$= 12\sqrt{3} - 8\sqrt{2} + 12\sqrt{3} - 12\sqrt{2}$

$= 24\sqrt{3} - 20\sqrt{2}$

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5(n) $(2\sqrt{7}+3\sqrt{5})(2\sqrt{7}-3\sqrt{5})$

$= 4\sqrt{49} \cancel{- 6\sqrt{35}} \cancel{+ 6\sqrt{35}} - 9\sqrt{25}$

×7 ×5

$= 28 - 45$

$= -17$

$2^3 = 8$

$\sqrt[3]{8} = 2$

$(\sqrt[3]{8})^3 = 8$

or $(2\sqrt{7})^2 - (3\sqrt{5})^2$

$= 4(7) - 9(5)$

$= -17$

$(\sqrt[2]{7})^2 = \sqrt{7}\sqrt{7}$

$= \sqrt{49}$

$= 7$

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