Solving Quadratic Equations

Feb 11/2011

Recall: To solve is to find the value(s) that satisfy the equation, or make it true.

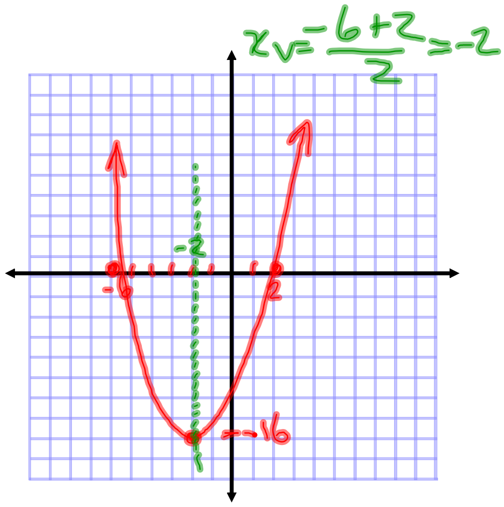
We often use this technique to find the zeroes, or roots, of a quadratic relation.

Ex.1 Solve $x^2 + 4x - 12 = 0$

$(x+6)(x-2)=0$

$x+6=0$ or $x-2=0$

$x=-6$ $\qquad$ $x=2$



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Factoring is not the only option:

Ex.2 Solve $x^2 - 6x - 27 = 0$ by completing the square.

$x^2-6x-27=0 \rightarrow (x-9)(x+3)=0$

$\underbrace{x^2-6x+9}-9-27=0$

$(x-3)^2-36=0$

$(x-3)^2=36$

$x-3=\pm\sqrt{36}$

$x-3=\pm 6$

$x-3=6$ $\qquad$ $x-3=-6$

$x=9$ $\qquad$ $x=-3$

$*$ $x^2=4$

$x=\pm\sqrt{4}$

$x=\pm 2$

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Ex.3 Solve $2x^2 - 5x - 1 = 0$ by completing the square.

$2x^2-5x-1=0$ $\qquad -\frac{5}{2}\times\frac{1}{2}=-\frac{5}{4}$

$2(x^2-\frac{5}{2}x)-1=0$ $\qquad (-\frac{5}{4})^2=\frac{25}{16}$

$2(\underbrace{x^2-\frac{5}{2}x+\frac{25}{16}}-\frac{25}{16})-1=0$

$2[(x-\frac{5}{4})^2-\frac{25}{16}]-1=0$ $\qquad \overset{1}{\cancel{2}}(-\frac{25}{\underset{8}{\cancel{16}}})$

$2(x-\frac{5}{4})^2-\frac{25}{8}-\frac{8}{8}=0$ $\qquad -1\times\frac{8}{8}=-\frac{8}{8}$

$2(x-\frac{5}{4})^2-\frac{33}{8}=0$

$\frac{1}{2}\times 2(x-\frac{5}{4})^2=\frac{33}{8}\times\frac{1}{2}$

$(x-\frac{5}{4})^2=\frac{33}{16}$

$x-\frac{5}{4}=\pm\sqrt{\frac{33}{16}} \rightarrow \frac{\sqrt{33}}{\sqrt{16}}$

$x-\frac{5}{4}=\pm\frac{\sqrt{33}}{4}$

$x=\pm\frac{\sqrt{33}}{4}+\frac{5}{4}$

$x=\frac{5}{4}\pm\frac{\sqrt{33}}{4}$

$x=\frac{5\pm\sqrt{33}}{4}$

both equally acceptable

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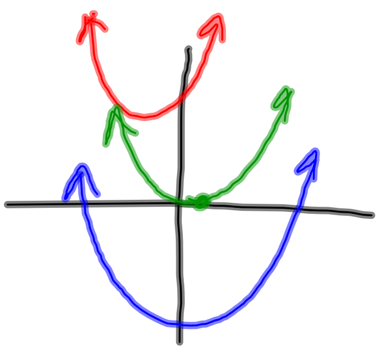
Recall: The general quadratic formula

Given $ax^2+by+c=0$, the solutions are:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

The number of real solutions can be determined from the discriminant, b^2-4ac:

$b^2-4ac>0$	two real solutions
$b^2-4ac=0$	one real solution
$b^2-4ac<0$	no real solutions



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Ex.4 Solve $2x^2 - 5x - 1 = 0$ using the general quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 2$
$b = -5$
$c = -1$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-1)}}{2(2)}$$

$$x = \frac{5 \pm \sqrt{25 + 8}}{4}$$

$$x = \frac{5 \pm \sqrt{33}}{4}$$

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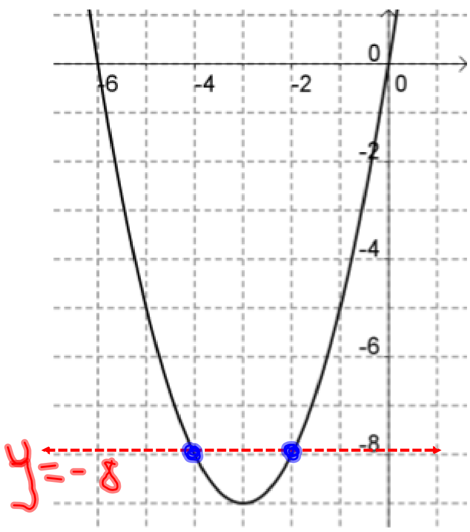
Given the relation $y = ax^2 + bx + c$, any of these methods can be used to solve for <u>any</u> value of y.

Consider $y = x^2 + 6x$, and solve for $y = -8$.

$$-8 = x^2 + 6x$$
$$0 = x^2 + 6x + 8$$
$$0 = (x+2)(x+4)$$

$x = -2$ OR $x = -4$



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Assigned Work:

p.128 # 2odd, 3ace, 4ace, 12ac, 13odd
p.130 # 17, 21, 23, 28ab, 29

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3(a) $\underbrace{x^2+6x}+4=0$

$\underbrace{x^2+6x+9}-9+4=0$

$(x+3)^2-5=0$

$(x+3)^2=5$

$x+3=\pm\sqrt{5}$

$x=-3\pm\sqrt{5}$

$x=-3+\sqrt{5}$ OR $x=-3-\sqrt{5}$

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4(a) $2x-6=-5x^2$

$$\frac{2}{5}\times\frac{1}{2}=\frac{2}{10}=\frac{1}{5}$$

$$\left(\frac{1}{5}\right)^2=\frac{1}{25}$$

$$5x^2+2x-6=0$$

$$5\left(x^2+\frac{2}{5}x\right)-6=0$$

$$5\left(x^2+\frac{2}{5}x+\frac{1}{25}-\frac{1}{25}\right)-6=0$$

$$5\left[\left(x+\frac{1}{5}\right)^2-\frac{1}{25}\right]-6=0$$

$$5\left(x+\frac{1}{5}\right)^2-\frac{\cancel{5}^{1}}{\cancel{25}_{5}}-6=0$$

$$5\left(x+\frac{1}{5}\right)^2-\frac{1}{5}-\frac{30}{5}=0$$

$$5\left(x+\frac{1}{5}\right)^2-\frac{31}{5}=0$$

$$\frac{1}{5}\times5\left(x+\frac{1}{5}\right)^2=\frac{31}{5}\times\frac{1}{5}$$

$$\left(x+\frac{1}{5}\right)^2=\frac{31}{25}$$

$$x+\frac{1}{5}=\pm\frac{\sqrt{31}}{5}$$

$$x=-\frac{1}{5}\pm\frac{\sqrt{31}}{5}$$

$$x=\frac{-1\pm\sqrt{31}}{5}$$

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4(a) $2x^2+8x+5=0$

$$2\left(x^2+4x\right)+5=0$$

$$2\left(x^2+4x+4-4\right)+5=0$$

$$2\left[(x+2)^2-4\right]+5=0$$

$$2(x+2)^2-8+5=0$$

skipped steps

$$2(x+2)^2=3$$

skipped

$$x+2=\pm\sqrt{\frac{3}{2}}$$

$$x=-2\pm\sqrt{\frac{3}{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$

$$x=-2\pm\frac{\sqrt{6}}{2}$$

$$x=-\frac{4}{2}\pm\frac{\sqrt{6}}{2}$$

$$x=\frac{-4\pm\sqrt{6}}{2}$$

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17.

$A = \frac{1}{2}bh$

$5 = \frac{1}{2}(h+2)(h)$

$10 = h^2 + 2h$

$0 = \underbrace{h^2 + 2h}{} - 10$

$0 = h^2 + 2h + 1 - 1 - 10$

$0 = (h+1)^2 - 11$

$(h+1)^2 = 11$

$h + 1 = \pm\sqrt{11}$

$h = -1 \pm \sqrt{11}$

$h = -1 + \sqrt{11}, \; h > 0$

$b = h + 2$

$= -1 + \sqrt{11} + 2$

$= 1 + \sqrt{11}$

$= 4.3$

$\therefore$ the base of the triangle is 4.3cm

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