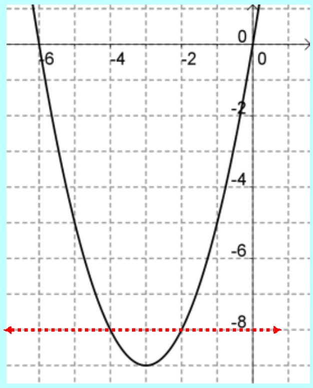
Intersection of Quadratics & Lines
(more solving quadratic equations)

Recall from last class:

Consider $y = x^2 + 6x$, and solve for $y = -8$.



In this example, we were actually solving for the intersection between the parabola and the horizontal straight line.

Solutions: (-4, -8) and (-2, -8)

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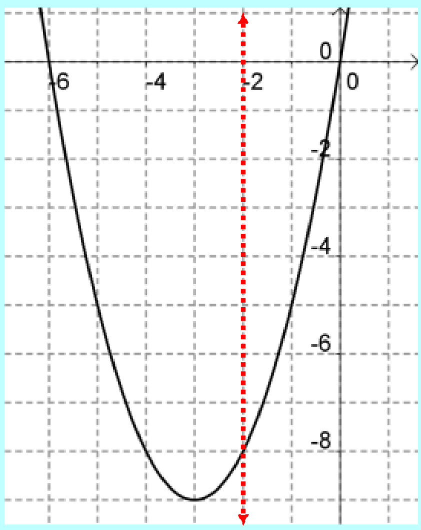
Intersection of Quadratics & Lines
(more solving quadratic equations)

Recall from last class:

Consider $y = x^2 + 6x$, and solve for $x = -2$.



In this example, we solve for the intersection between the parabola and the vertical straight line.

Solution: (-2, -8)

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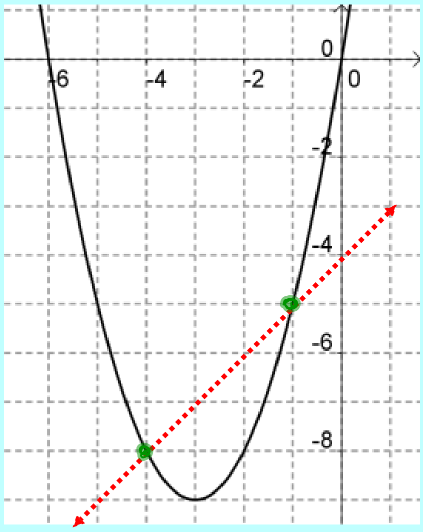
Intersection of Quadratics & Lines

(more solving quadratic equations)

Recall from last class:

Consider $y = x^2 + 6x$, and solve for $y = x - 4$.



In this example, we solve for the intersection between the parabola and the given straight line.

Solutions: (-4, -8) and (-1, -5)

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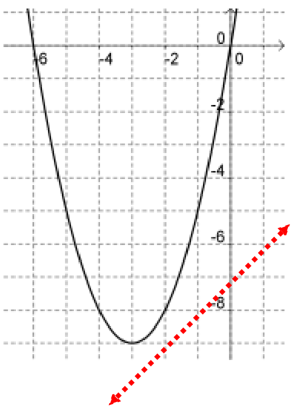
Intersection of Quadratics & Lines

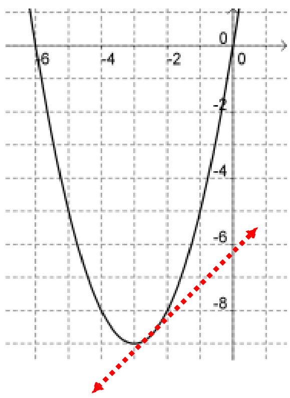
Feb 14/2011

(more solving quadratic equations)

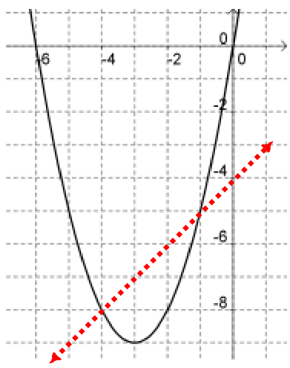
A linear-quadratic system will have zero, one, or two solutions.



No Solution



One Solution (tangent line)



Two Solutions (secant line)

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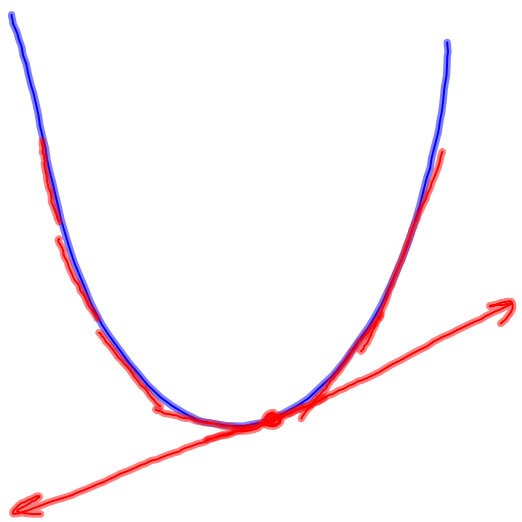

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Recall: To <u>solve</u> an equation is to find the value(s) for the variables that satisfy the equation (i.e., make it true)

Given a quadratic relation, $y = Ax^2 + Bx + C$

and a linear relation, $y = mx + b_1$

the solution will be the point(s) where the parabola and straight line intersect.

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$y = Ax^2 + Bx + C$ (1) $\qquad$ $y = mx + b_1$ (2)

Solve the <u>system of equations</u> using the fact that $y = y$

$$y = y$$
$$Ax^2 + Bx + C = mx + b_1$$
$$\vdots \quad \text{rearrange}$$
$$ax^2 + bx + c = 0$$

Sub the x-values from the solution(s) into <u>either</u> original relation to find the corresponding y-values.

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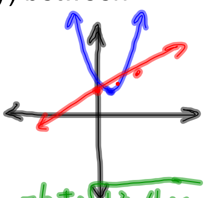
Ex.1 Find the points of intersection (if any) between
① $y = 2(x-1)^2 + 2$ and $y = x + 2$. ②



$$2(x-1)^2 + 2 = x + 2$$
$$2(x^2 - 2x + 1) + 2 = x + 2$$
$$2x^2 - 4x + 2 + 2 = x + 2$$
$$2x^2 - 5x + 2 = 0$$
$$2x^2 - x - 4x + 2 = 0$$
$$x(2x-1) - 2(2x-1) = 0$$
$$(2x-1)(x-2) = 0$$
$$x = \tfrac{1}{2} \text{ or } x = 2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$b^2 - 4ac = 25 - 4(2)(2)$$
$$= 25 - 16$$
$$= 9$$

S: -5
P: 4
I: $-1, -4$

Sub $x = \frac{1}{2}$ into ②
$$y = \tfrac{1}{2} + 2$$
$$= 2.5$$
$$= \tfrac{5}{2}$$

Sub $x = 2$ into ②
$$y = 2 + 2$$
$$= 4$$

$\therefore$ Solutions are $\left(\frac{1}{2}, \frac{5}{2}\right)$ and $(2, 4)$

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Ex.2 Determine the equations of the lines that have a slope of 2 that intersect $y = x(6 - x)$
(a) once
(b) twice
(c) never

$y = 2x + b$

$x(6-x) = 2x + b$
$6x - x^2 = 2x + b$
$0 = x^2 - 4x + b$

(a) $b^2 - 4ac = 0$
$(-4)^2 - 4(1)(b) = 0$
$16 - 4b = 0$
$16 = 4b$
$b = 4$

$\therefore$ for one solution,
$y = 2x + 4$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(b) 2 sol'n : $b^2 - 4ac > 0$
$16 - 4b > 0$

try $b = 5$, $16 - 4(5) = 16 - 20$
$= -4 \rightarrow$ no solution

try $b = 3$, $16 - 4(3) = 16 - 12$
$= 4 \rightarrow 2$ solutions

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Assigned Work:

worksheet

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#

$y = -x + 6 \qquad y = -2x^2 + 3x - 2$

$-x + 6 = -2x^2 + 3x - 2$

$\frac{2x^2}{2} - \frac{4x}{2} + \frac{8}{2} = \frac{0}{2}$

$0 = -2x^2 + 4x - 8$

$x^2 - 2x + 4 = 0$

$\therefore$ no solution

$b^2 - 4ac$

$= (-2)^2 - 4(1)(4)$

$= 4 - 16$

< 0

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5. y-int $= -3 \rightarrow (0,-3)$

parabola	line
$(2,1)$	$(4,9)$
$(4,-11)$	$(0,-3)$
$(0,-3)$	

$y = ax^2 + bx + c$

$y = ax^2 + bx - 3$

sub $(2,1)$

$1 = 4a + 2b - 3$

$0 = 4a + 2b - 4$ ①

sub $(4,-11)$

$-11 = 16a + 4b - 3$

$0 = 16a + 4b + 8$ ②

①×2: $0 = 8a + 4b - 8$

$-\quad 0 = 8a \quad + 16$

$-16 = 8a$

$\boxed{a = -2}$

$y = -2x^2 + 6x - 3$

$y = mx + k$

$y = mx - 3$

$9 = 4m - 3$

$12 = 4m$

$\boxed{m = 3}$

$y = 3x - 3$

sub $a = -2$ into ①

$0 = 4(-2) + 2b - 4$

$0 = -8 + 2b - 4$

$12 = 2b$

$\boxed{b = 6}$

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3. $(1, 11)$
$x \quad y$

$$y_1 = 3(x-h)^2 + 8$$

$$11 = 3(1-h)^2 + 8$$

$$\frac{3}{3} = \frac{3}{3}(1-h)^2$$

$$1 = (1-h)^2$$

$$\pm 1 = 1 - h$$

$$\frac{-1}{-1} \frac{\pm 1}{-1} = \frac{-h}{-1}$$

$$1 \pm 1 = h$$

$$h = 0 \text{ or } h = 2$$

$$y_2 = mx + 17$$

$$11 = m + 17$$

$$\boxed{m = -6}$$

$$y_2 = -6x + 17$$

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