Transformations:

1) State the transformations) that each function has undergone. Then state the transformations in function and in mapping notation.
a) $y=-\frac{1}{4} \sqrt{3(x-7)}$, base function: $f(x)=\sqrt{x}$
V. compression by a factor of 4 reflection along the $x$-axis $y=-\frac{1}{2} f(3(x-7))$
H. compression by a factor of $3 \quad(x, y) \rightarrow\left(\frac{1}{3} x+7,-\frac{1}{2} y\right)$
$H$. translation to the right 7 units
b) $y=3(4-x)^{3}-6$, base function: $f(x)=x^{3}$

$$
\begin{aligned}
& y=3(-x+4)^{3}-6 \\
& y=3[-1(x-4)]^{3}-6
\end{aligned}
$$

$V$. stretch by a factor of 3
reflection along the $y$-axis
H. shift right 4 units

$$
y=3 f(-(x-4))-6
$$

v. translation 6 units down
c) $y=-3(4)^{x-1}$, compare to $f(x)=(4)^{x}$
reflection along the $x$-axis
$V$. stretch by a factor of 3
H. translation right 1 unit
fruct. notation: $y=-3 f(x-1)$
mapping notation: $(x, y) \rightarrow(x+1,-3 y)$
d) $y=\log (-x)$, compare to $f(x)=\log (x)$
reflection along the $y$-axis
fact. notation: $y=f(-x)$

$$
\text { mapping notation: }(x, y) \rightarrow(-x, y)
$$

e) $y=-2 \cos \frac{1}{3}\left(\theta-\frac{\pi}{2}\right)+1$, compare to $f(\theta)=\cos (\theta)$
reflection along the $x$-axis
$v$. stretch by a factor of 2
H. stretch by a factor of 3
V. shift up init
frit notation: $y=-2 f\left(\frac{1}{3}\left(\theta-\frac{\pi}{2}\right)\right)+1$
mapping notation: $\left.(x, y) \rightarrow\left(3 x+\frac{\pi}{2}\right)-2 y+1\right)$
2) The graph of $f(x)=x^{4}$ is horizontally stretched by a factor of 2 , reflected in the $y$-axis, and shifted up 5 units. Find the equation of the transformed function.

$$
\begin{aligned}
& \text { H. stretch by a factor of } 2 \\
& \text { reflected in the } y \text {-axis } \\
& \text { shifted up } 5 \text { units } \\
& \qquad y=\left(-\frac{1}{2}(x)\right)^{4}+5 \quad \text { note this would be the } \\
& \qquad y=\left(-\frac{1}{2} x\right)^{4}+5 \quad \text { same graph as } \\
& y=\frac{1}{16} x^{4}+5
\end{aligned}
$$

Inverses:
3) Determine the inverse of each of the following. State if the invese is not a function and state any restrictions.

$$
\begin{aligned}
& \text { a) } y=x^{2}-10 \\
& \text { inverse: } \quad x=y^{2}-10 \\
& x+10=y^{2} \\
& \pm \sqrt{x+10}=y \\
& \text { not a duct. } \\
& D_{\text {inverse }}:\{x \mid x \geqslant-10, x \in \mathbb{R}\} \\
& \text { extra: for the inverse to be } \\
& \text { a fruit } \begin{aligned}
& D_{\text {original }}:\{|x| x \geqslant 0, x \in \mathbb{R}\} \\
& \text { or } x \leqslant 0
\end{aligned} \\
& \text { b) } \begin{array}{l}
f(x)=2 x+1 \\
f: \quad y=2 x+1
\end{array} \\
& \text { inverse: } \quad x=2 y+1 \\
& x-1=2 y \\
& \frac{1}{2} x-\frac{1}{2}=y \\
& \frac{1}{2} x-\frac{1}{2}=f^{-1}(x) \\
& D_{f}:\{\{x \mid x \in \mathbb{R}\} \\
& \because \text { inverse is } \\
& \text { a fut. }
\end{aligned}
$$

$$
\begin{aligned}
& \text { c) } y=\frac{1}{x+3}-1 \\
& \text { d) } y=\log _{2} x \\
& \text { inverse: } \quad x=\frac{1}{y+3}-1 \\
& x+1=\frac{1}{y+3} \\
& y+3=\frac{1}{x+1} \\
& y=\frac{1}{x+1}-3 \\
& f^{-1}(x)=\frac{1}{x+1}-3 \\
& \text { inverse: } x=\log _{2} y \\
& \Leftrightarrow \quad 2^{x}=y \\
& 2^{x}=f^{-1}(x) \\
& D_{f^{-1}(x)}:\{x \mid x \neq-1, x \in \mathbb{R}\} \\
& D_{f^{n}}:\{x \mid x \in \mathbb{R}\} \\
& \text { e) } y=10^{x-4}+7 \\
& \text { inverse: } x=10^{y-4}+7 \\
& x-7=10^{y-4} \\
& \Leftrightarrow y-4=\log _{10}(x-7) \\
& y=\log _{10}(x-7)+4 \\
& f^{-1}(x)=\log _{10}(x-7)+4 \\
& \text { The inverse is a frat.' } \\
& D_{f^{--}}:\{x \mid x>7, x \in \mathbb{R}\}
\end{aligned}
$$

4) Sketch the inverse of the following functions on the same axes.

5) Sketch $f(x)=(x-2)^{2}$ and its inverse. What would the domain have to be so that the inverse is a function?

$$
f(x)=(x-2)^{2}
$$



$$
\begin{aligned}
D_{f}: & \{x \mid x \geqslant 0, x \in \mathbb{R}\} \\
& \text { so that inverse is a frict. } \\
D_{f}: & \{x \mid x=0, x \in \mathbb{R}\} \\
& \text { so that inverse is a fret. }
\end{aligned}
$$

Functions, properties and their graphs:
6) Given the graph of $y=f(x)$ shown on the right, state the intervals of $x$ for which
(a) the function is decreasing
(b) the function is concave up
a) fruct is decr.: $-3<x<1$
b) frat is conc. down: $x<-1$

7) For the following graph, state the number of turning points, the number of inflection points, and the intervals of increase and decrease:


3 turning pts (2 max and ore min.)
4 points of inflections
incr. $x<-1.5$ or $0<x<1.3$
decr. $-1.5<x<0$ or $1.3<x<3$ or $x>3$
8) Which graph best matches the equation: $y=\frac{2 x^{2}+x-3}{x^{2}-4 x+3}$ ?



8)

$$
y=\frac{2 x^{2}+x-3}{x^{2}-4 x+3}
$$

8) $y=\frac{2 x^{2}+x-3}{x^{2}-4 x+3}$

$$
\begin{aligned}
& P=-6 \quad 3,-2 \\
& S=1 \quad 3,-3 \\
& =3
\end{aligned}
$$

$$
s=-4
$$

factor to determine zeros, $V A$, holes, H4/OA

$$
\begin{aligned}
& y=\frac{(2 x+3)(x-1)}{(x-3)(x-1)} \\
& y=\frac{2 x+3}{x-3} ; \quad x \neq 1
\end{aligned}
$$

$$
\therefore \text { hole at } x=1
$$

$$
12+x=1
$$

$$
y=\frac{2(1)+3}{1-3}
$$

$$
y=-\frac{5}{2}
$$

$$
\therefore \text { hole }\left(1, \frac{-5}{2}\right)
$$

$$
V A: \text { let } x-3=0
$$

$$
x=3
$$

$$
\text { zero: let } 2 x+3=0
$$

$$
2 x=-3
$$

$$
x=\frac{-3}{2}
$$

HA equal degrees
$\therefore$ divide coff.

$$
y=2
$$

These properties match the $3^{\text {rd }}$ graph.
9) Sketch each of the following functions labelling: intercepts and asymptotes and stating the domain.
a) $y=-(x+2)^{3}$
$\rightarrow$ special cubic, no asymptotes.
$\rightarrow$-re lead coff
$\therefore$ as $x \rightarrow \infty, y \rightarrow-\infty$
shifted left 2 units
$\rightarrow$ shifted left 2 units
$\rightarrow$ zero -2 , pt. of inflection

$$
\begin{array}{cl}
x \text {-int (Let } y=0) & y \text {-int. (let } x=0) \\
0=-(x+2)^{3} & y=-(2)^{3} \\
0=(x+2)^{3} & y=-8 \\
\sqrt[3]{0}=x+2 &
\end{array}
$$


$D:\{x \mid x \in \mathbb{R}\}$
$0=x+2$
$-2=x$
b) $y=(x-3)^{2}\left(x^{2}-4 x-7\right)$

$$
\begin{aligned}
& \rightarrow \text { quartic, no asymptotes } \\
& \rightarrow \text { tee lead coff } \\
& \therefore \text { as } x \rightarrow \infty, y \rightarrow \infty \\
& \rightarrow \text { DR at } x=3 \text {, turning } p^{+} . \\
& x \text {-int (let } y=0) \\
& (x-3)^{2}=0 \quad x^{2}-4 x-7=0 \\
& x=3 \quad x=\frac{4 \pm \sqrt{16+28}}{2} \\
& x=5.32 \\
& x=-1.32
\end{aligned}
$$

$$
y=(-3)^{2}(-7)
$$

$$
y=-63
$$

$$
D\{x \mid x \in \mathbb{R}\}
$$

c) $y=x^{4}+2 x^{3}+x^{2}+2 x$

$$
\begin{aligned}
& \rightarrow \text { quartic, no asymptotes } \\
& \rightarrow \text { tree lead coff. } \\
& \quad . \text { as } x \rightarrow \infty, y \rightarrow \infty
\end{aligned}
$$

$x$-int. (let $y=0$ )
$0=x\left(x^{3}+2 x^{2}+x+2\right)$
$0=x\left[x^{2}(x+2)+1(x+2)\right]$
$0=x(x+2)\left(x^{2}+1\right)$

$$
x=0, \quad x=-2 \quad \text { no real soil }{ }^{2}
$$

$D\{x \mid x \in \mathbb{R}\}$


$$
y \text {-int }(l+x=0)
$$

$$
y=0
$$

$D:\{x \mid x \in \mathbb{R}\}$
d) $f(x)=\frac{x^{2}+3}{x+4}$
d) $f(x)=\frac{x^{2}+3}{x+4}$

$$
\begin{aligned}
& \rightarrow \text { rational frat } \\
& \rightarrow \text { dey numerator }>\text { dey denom. } \\
& \text { by }: \therefore O A
\end{aligned}
$$

$$
\rightarrow \text { tue lead coif. }
$$

$$
\therefore \text { ends in } Q I
$$

$$
x \text {-int (let } y=0)
$$

$$
0=\frac{x^{2}+3}{x+4}
$$

$$
\Rightarrow \text { numerator }=0
$$

$$
x^{2}+3=0
$$

no real sol

$$
\therefore \text { no saint }
$$

$$
y \text {-int (let } x=0)
$$

$$
y=\frac{3}{4}
$$

$$
V A: x=-4
$$

e) $f(x)=\log (x+2)$

$$
\begin{aligned}
& \text { qt) } f(x)=\log (x+2) \\
& \rightarrow \text { Logarithmic fuck, VA } \\
& \text { VA let } x+2=0 \\
& \rightarrow \text { whiffed left units } \\
& x \text {-int (let } y=0) \\
& 0=\log (x+2) \\
& \Leftrightarrow 10^{\circ}=x+2 \\
& 1=x+2 \\
& -1=x \\
& y \text {-int (let } x=0 \text { ) } \\
& y=\log 2 \\
& y=0.3010 \\
& D \cdot\{x \mid x>-2, x \in R\}
\end{aligned}
$$

f) $f(x)=3 \cdot 5^{-x}+4$

$$
\begin{aligned}
& \rightarrow \text { exponential fuck, HA } \\
& \rightarrow \text { V. stretch by a tractor of } 3 \\
& \begin{array}{l}
\rightarrow \text { reflection along } y \text {-axis } \\
\rightarrow \text { V.shift up } 4 u n i t s
\end{array} \\
& \begin{array}{c}
x \text {-int (let } y=0 \text { ) } \\
0=3 \cdot 5^{-x}+4
\end{array} \\
& -4=3 \cdot 5^{-x} \\
& f(0)=7 \\
& -\frac{4}{3}=5^{-x} \quad\left(n o s .1^{4}\right) \\
& \Leftrightarrow \log _{5}-\frac{4}{3}=-x \ll \text { check } \\
& \frac{\log _{\log } \frac{-4}{3}}{\log _{\text {no }}}=-x \\
& \text { D: }\{x \mid x \in \mathbb{R}\}
\end{aligned}
$$

g) $f(x)=\frac{10-10 x}{(x-4)^{2}}$

$$
\begin{aligned}
& \rightarrow \text { rational fact } \\
& \rightarrow \operatorname{deg}_{\therefore \text { numerator }}^{<} \text {dey if evan } \\
& \rightarrow \text {-vt lead conf } \\
& \therefore \text { ends in QIV } \\
& x \text {-int } \quad V A \text { let }(x-4)^{2}=0 \\
& \text { let }-10 x+10=0 \\
& 10=10 x \\
& 1=x \\
& y \text {-int let } x=0 \\
& f(0)=\frac{10}{(-4)^{2}} \\
& f(0)=\frac{5}{8} \\
& x=4 \quad D R \\
& \therefore \text { arrows in } \\
& \text { same direction } \\
& \text { HA } \quad \underset{\sim}{\sim} \\
& x=4 \\
& D:\{x \mid x \neq 4, x \in \mathbb{R}\}
\end{aligned}
$$

h) $f(x)=\frac{x^{2}+4}{x^{2}-4}$
h) $f(x)=\frac{x^{2}+4}{x^{2}-4}$
$\rightarrow$ rational fact
$\rightarrow \operatorname{deg} \begin{gathered}\text { numerator } \\ \therefore \text { HA } \\ =\text { dey denominator }\end{gathered}$
$\checkmark A$ let $x^{2}-4=0$
$\rightarrow$ the land cerf.
$\therefore$ ends Q I
$x$ int let $x^{2}+4=0$ no sol 18
$f(0)=-1$

$$
x= \pm 2
$$

$$
\text { HA } \div \text { coeff } \because \text { deg equal }
$$

$$
\begin{array}{r}
x-\operatorname{int} \text { let } x^{2}+ \\
y \text {-int let } x=0 \\
f(0)=\frac{4}{-4} \\
f(0)=-1
\end{array}
$$


i) $y=2 \sin \left(\theta-\frac{\pi}{2}\right)+1$


$$
D:\{x \mid x \in \mathbb{R}\}
$$

$$
\begin{aligned}
& \rightarrow \text { sine fact } \\
& x \operatorname{int}(\text { let } y=0) \\
& 0=2 \sin \left(\theta-\frac{\pi}{2}\right)+1 \\
& \rightarrow \text { Amp: } 2 \quad-1=2 \sin \left(\theta-\frac{\pi}{2}\right) \\
& \text { ps: } \frac{\pi}{2} \\
& -\frac{1}{2}=\sin \left(\theta-\frac{\pi}{2}\right) \\
& v \text {. isp }=1 \\
& \text { let } B=\theta-\frac{\pi}{2} \\
& y \text {-int }(\operatorname{let} \theta=0) \\
& \frac{1}{2}=\sin B_{r} \\
& y=2 \sin \left(-\frac{\pi}{2}\right)+1 \\
& \frac{\pi}{6}=B_{r} \\
& B_{1}=\pi+\frac{\pi}{6} \\
& B_{2}=2 \pi-\frac{\pi}{6} \\
& y=-2+1 \\
& \theta_{1}-\frac{\pi}{2}=\frac{7 \pi}{6} \\
& \theta_{2}-\frac{\pi}{2}=\frac{11 \pi}{6} \\
& \theta_{1}=\frac{7 \pi}{6}+\frac{\pi}{2} \\
& \theta_{2}=\frac{11 \pi}{6}+\frac{\pi}{2} \\
& \theta_{1}=\frac{10 \pi}{6} \\
& \theta_{2}=\frac{14 \pi}{6} \\
& \theta_{1}=\frac{5 \pi}{3} \\
& \theta_{i}=\frac{7 \pi}{3} \\
& \theta_{3}=\frac{5 \pi}{3} \pm 2 n \pi \quad \theta_{2}=\frac{7 \pi}{3} \pm 2 n \pi
\end{aligned}
$$

j) $y=\tan 2 \theta-1$

$$
\begin{aligned}
& \text { j) } y=\tan 2 \theta-1 \\
& \rightarrow \text { tangent fret, VA } \begin{aligned}
x-\operatorname{int} & (6+t y=0) \\
0 & =\tan 2 \theta-1
\end{aligned} \\
& \rightarrow \text { Period }=\frac{\pi}{2} \\
& 1=\tan 2 \theta \\
& \text { V.shiff down } 1 \\
& \text { Let } B=2 \theta \\
& y \text {-int (let } \theta=0 \text { ) } \\
& 1=\tan B \\
& y=\tan 0-1 \\
& \frac{\pi}{4}=B_{1} \quad B_{2}=\frac{5 \pi}{4} \\
& y=-1 \\
& \frac{\pi}{8}=\theta_{1} \quad \theta_{2}=\frac{5 \pi}{8} \\
& \begin{array}{r}
\frac{\pi}{8} \pm \frac{n \pi}{2}=\theta_{3} \quad \theta_{4}=\frac{\frac{5 \pi}{8}}{8} \pm \frac{n \pi}{2} \\
V_{A} \text { for } y=\tan \theta
\end{array} \\
& \text { VA } x=\frac{\pi}{2} \\
& 4 \text { compression } \\
& x=\frac{\pi}{4} \\
& D:\left\{x \left\lvert\, x \neq \frac{\pi}{4}+n \pi\right., n \in \mathbb{Z}, x \in \mathbb{R}\right\}
\end{aligned}
$$

k) $y=0.5 \sec \left(\theta+\frac{\pi}{4}\right)$

$$
\begin{aligned}
& \rightarrow \text { secant fret. } \\
& x \text {-int. (let } y=0) \\
& \rightarrow \text { reciprocal of cosine } \\
& 0=0.5 \sec \left(\theta+\frac{\pi}{4}\right) \\
& \rightarrow \text { Amp not applicable } \\
& 0=\sec \left(\theta+\frac{\pi}{4}\right) \\
& \text { H. shot left } \frac{5}{4} \\
& 0=\frac{1}{\cos \left(\theta+\frac{\pi}{4}\right)} \\
& y \text {-int (let } \theta=0 \text { ) } \\
& y=0.5 \sec \left(\frac{\pi}{4}\right) \\
& \cos \left(\theta+\frac{\pi}{4}\right)=\text { under. } \\
& y=0.5 \cdot \frac{1}{\cos \frac{5}{4}} \\
& \text { not possible. } \\
& y=0.5 \frac{\sqrt{2}}{1} \\
& \checkmark A \text { for } y=\sec \theta \\
& \text { when } \theta=\frac{\pi}{2} \pm n \pi \\
& y=\frac{\sqrt{2}}{2} \\
& \text { (zeus of } y=\cos \theta \text { ) } \\
& \text { H. shift left } \frac{\pi}{4} \\
& y=0.7071 \\
& \therefore V A \quad \theta=\frac{\pi}{2}-\frac{\pi}{4} \pm n \pi \\
& \theta=\frac{\pi}{4} \pm n \pi
\end{aligned}
$$


$D:\left\{x \left\lvert\, x \neq \frac{\pi}{4} \pm n \pi\right., n \in \mathbb{Z}, x \in \mathbb{R}\right\}$
10) Analyze and sketch the following, using intercepts, asymptotes, and end behaviours:

$$
\begin{aligned}
& y=\frac{3 x^{3}+10 x^{2}+3 x}{x^{2}+5 x+6} \\
& \text { zeros: let } 3 x^{3}+10 x^{2}+3 x=0 \quad \text { let } x^{2}+5 x+6=0 \\
& x\left(3 x^{2}+10 x+3\right)=0 \quad(x+3)(x+2)=0 \\
& x(3 x+1)(x+3)=0 \\
& x=0, x=\frac{-1}{3}, x=-3 \\
& y \text {-int let } x=0 \\
& y=\frac{0}{6} \\
& y=0 \\
& x=-3, x=-2 \\
& \hat{T} \quad V A \\
& \therefore \text { hole } \\
& \text { nole: } y=\frac{x(3 x+1)}{x+2} \\
& y=\frac{-3(-8)}{-1} \\
& \text { 0.4: } \frac{3 x-5}{x^{2}+5 x+6} \sqrt{3 x^{3}+10 x^{2}+3 x+0} \\
& y=-24 \\
& \frac{-\left(3 x^{3}+15 x^{2}\right)}{-5 x^{2}+3 x+0} \downarrow \\
& \frac{-\left(-5 x^{2}-25 x-30\right)}{28 x+30}
\end{aligned}
$$

11) A polynomial of degree 5 has a negative leading coefficient.
a) How many turning points could the polynomial have?
b) How many zeros could the function have?
c) Describe the end behaviour.
d) Sketch two possible graphs, each passing through the point $(1,-2)$.

## 1 poly of dey 5 <br> $\Rightarrow n=5$

a) max \# of turning pts $\begin{aligned} & =n-1 \\ & =4\end{aligned}$
$\begin{aligned} & \therefore 0,2 \text { or } 4 \text { furning pts } \\ & \text { max of zeros }=n \\ &=5\end{aligned}$ odd deg.
It zeros $1,2,3,4$ or 5
c) -ve lead coeff $\therefore$ as $x \rightarrow \infty, y \rightarrow-\infty$ odd deg as $x \rightarrow-\infty, y \rightarrow \infty$

many answers

Symmetry:
12) Determine whether each of the following functions is even, odd, or neither. Justify your answer.
a)

a)


$f(-x)=-f(x) \therefore$ odd fret.
b)

b)


$f(x) \neq f(-x), \quad f(-x) \neq-f(x) \therefore$ neither odd nor
c)

c)

d) $f(x)=3 x^{2}+4$
d) $f(x)=3 x^{2}+4$
$f(-x)=3(-x)^{2}+4$
$f(-x)=3 x^{2}+4$
$f(-x)=f(x)$
$\therefore$ fret is even
e) $f(x)=-3 x^{3}+x$
e) $f(x)=-3 x^{3}+x$
$f(-x)=-3(-x)^{3}+(-x)$
$f(-x)=3 x^{3}-x$
$-f(x)=-\left(-3 x^{3}+x\right)$
$-f(x)=3 x^{3}-x$
$-f(x)=f(-x)$
$\therefore$ odd fact.
f) $f(x)=\tan x$
g) $y=3^{x}+1$
f) $f(x)=\tan x$, assume $x \in Q I$

$$
\begin{array}{ll}
f(-x)=\tan (-x), & -x \in Q \mathbb{V} \\
f(-x)=-\tan x & \text { tan ratio is } \\
\text {-re in QIV }
\end{array}
$$

g) $\begin{aligned} y & =3^{x}+1 \\ \text { let } y & =f(x)\end{aligned}$ $f(x)=3^{x}+1$
$f(-x)=3^{-x}+1$
$f(-x) \neq f(x)$
$-f(x)=-\left(3^{x}+1\right)$
$-f(x)=-\tan x$
$-f(x)=f(-x) \therefore$ odd fact $\quad \begin{aligned} & -f(x)=-\left(3^{x}+1\right) \\ & -f^{x}-1\end{aligned}$
h) $f(x)=3 \log x-1$
i) $y=\frac{1}{x^{2}-4}$
h) $\begin{aligned} f(x) & =3 \log x-1 \\ f(-x) & =3 \log (-x)-1\end{aligned}$
i) $y=\frac{1}{x^{2}-4}$
$-f(x)=-(3 \log x-1)$
$-f(x)=-3 \log x+1$
$f(-x) \neq f(x)$
$f(-x) \neq-f(x)$
let $y=f(x)$.
$f(-x)=\frac{1}{(-x)^{2}-4}$
$f(-x)=\frac{1}{x^{2}-4}$
$f(-x)=f(x)$
$\therefore$ fact neither odd nor even
$\therefore$ frat is even
j) $f(x)=2^{x}+2^{-x}$
k) $f(x)=\frac{\sin x}{x^{2}-4}$
(use combinations of functions to justify)

$$
\text { j) } \begin{aligned}
& f(x)=2^{x}+2^{-x} \\
& f(-x)=2^{-x}+2^{-(-x)} \\
& f(-x)=2^{-x}+2^{x} \\
& \because \text { addition is commutative } \\
& f(-x)=2^{x}+2^{-x} \\
& f(-x)=f(x)
\end{aligned}
$$

$$
\text { k) } f(x)=\frac{\sin x}{x^{2}-4}
$$

$$
y=\sin x \quad \text { is an odd fact }
$$

$$
y=x^{2}-4 \text { is an even foot. }
$$

The quotient of an
odd and an even
fact is odd

1) $f(x)=x \cdot \log x$
(use combinations of functions to justify)

$$
\begin{aligned}
& \text { e) } f(x)=x \cdot \log x \\
& y=x \text { is an odd fact } \\
& y=\log x \text { is neither odd } \\
& \text { nor even } \\
& \text { The product of an } \\
& \text { odd and a neither } \\
& \text { is a neither. }
\end{aligned}
$$

## Rates of Change:

13) The position in kilometres of a particle at $t$ hours is given by $d(t)=t^{3}-12 t^{2}+34 t+75$, where $t \geq 0$.
a) What is the initial position of the particle?
b) What is the particle's average velocity from 3 hours to 5 hours?
c) What is the particle's instantaneous velocity at 7 hours?
a) initial position $\Rightarrow t=0$

$$
\begin{aligned}
& d(0)=75 \\
& \text { The initial position is } 75 \mathrm{~km} \text {. }
\end{aligned}
$$

b) avg. vel'y $=\operatorname{avg} . R_{0} C$ of $d(t)$

$$
\text { avg. vel'y }=\frac{d(5)-d(3)}{5-3}
$$

$$
\text { arg. very }=\frac{70-96}{2}
$$

$$
\text { avg. rel' } y=-13
$$

The pteles aug. vel'y from 3 hrs. to 5 hrs
is $-13 \mathrm{~km} / \mathrm{hr}$.
or The position is decs. at an avg. $R_{0} C$
or of $13 \mathrm{~km} / \mathrm{hr}$.
c)

14) Find the slope of the secant of $y=2^{x}-3$ that passes through the points where $x=-3$ and $x=1$.

$$
\begin{aligned}
y & =2^{x}-3 \\
m_{\text {sec }} & =\frac{\Delta y}{\Delta x} \\
& =\frac{\left.y\right|_{x=1}-\left.y\right|_{x=-3}}{1-(-3)} \\
& =\frac{-1+2.875}{4} \\
& =0.47 .
\end{aligned}
$$

15) The concentration of medicine in a patient's bloodstream is given by $C(t)=\frac{0.4 t}{(0.3 t+2)^{3}}, t \geq 0$, where $C$ is measured in milligrams per cubic centimetre and $t$ is the time in hours after the medicine was taken. Determine:
a) the concentration in the bloodstream 3 hours after the medicine was taken.
b) the average rate at which the concentration is decreasing from 4 hours after taking the medicine to 7 hours after taking the medicine.
c) the instantaneous rate of change for the concentration 2 hours after the medicine was taken. Interpret the meaning of your answer.
a) $\quad C(3)=\frac{0.4(3)}{[(0.3)(3)+2]^{3}}$

$$
C(3) \doteq 0.0492
$$

$$
\text { After } 3 \mathrm{hrs} \text { Theconc. is } 0.0492 \mathrm{mg} / \mathrm{cm}^{3}
$$

b) $\arg R_{0} C=\frac{\Delta C(t)}{\Delta t}$
avg. $R_{0} C=\frac{C(7)-C(4)}{7-4}$
avg. $R_{0} C=\frac{0.0406-0.0488}{3}$
avg. $R_{0} C \doteq-0.0027$
The conc. is deco. at an avg. ROC of $0.0027 \mathrm{mg} / \mathrm{cm}^{3} / \mathrm{hr}$.
c)

16) Complete the table


Solving Equations/inequalities:
17) Determine the solutions) of:
a) $349=7(1.49)^{x}$
) a) $\frac{349}{7}=\frac{7(1.49)^{x}}{7}$ $49.8571=1.49^{x}$
$\begin{aligned} \Leftrightarrow \log _{1.49} 49.8571 & \equiv x \\ \frac{\log 49.8571}{} 1.49 & =x \\ 9.80 & \doteq x\end{aligned}$
b) $\log _{2} 8=3 \log _{2} x-\log _{2} 3$

$$
\text { b) } \begin{aligned}
\log _{2} 8 & =3 \log _{2} x-\log _{2} 3 \\
\log _{2} 8 & =\log _{2} x^{3}-\log _{2} 3 \\
\log _{2} 8 & =\log _{2}\left(\frac{x^{3}}{3}\right) \\
8 & =\frac{x^{3}}{3} \\
\therefore \quad 24 & =x^{3} \\
\sqrt[3]{24} & =x \\
2.88 \quad & \doteq x, \quad x>0
\end{aligned}
$$

c) $7^{x}=3^{x^{2}-1}$

$$
\begin{array}{ll}
\Leftrightarrow \log _{3} 7^{x}=x^{2}-1 \quad \rightarrow \text { or } & \therefore \log 7^{x}=\log 3^{x^{2}-1} \\
x\left(\log _{3} 7\right)=x^{2}-1 & x \log ^{7}=\left(x^{2}-1\right) \log 3 \\
1.7712 x \doteq x^{2}-1 & 0.8451 x \doteq 0.4771 x^{2}-0.4771 \\
0 \doteq x^{2}-1.7712 x-1 & 0 \doteq 0.4771 x^{2}-0.8451 x \\
x=\frac{1.7712 \pm \sqrt{(-1.7712)^{2}-4(-1)}}{2} & x \doteq \frac{0.8451 \pm \sqrt{(-0.8451)^{2}-4(-0.4771)^{2}}}{2(0.4771)} \\
x \doteq 2.22 \text { or } x \doteq-0.45 & x=2.22 \quad \text { or } x \doteq-0.45
\end{array}
$$

d) $x^{3}-6 x^{2}+5 x+12>0$

$$
x^{3}-6 x^{2}+5 x+12>0
$$

$$
\text { let } \begin{aligned}
P(x) & =x^{3}-6 x^{2}+5 x+12 \\
P(-1) & =0 \therefore x+1 \text { is a factor }
\end{aligned}
$$

$$
-1 \frac{1}{\downarrow} \frac{-6}{1} \frac{-1}{1} \frac{5}{-7} \quad \frac{12}{12} \quad \frac{-12}{0}
$$

$\therefore(x+1)\left(x^{2}-7 x+12\right)>0$

when $-1<x<3$ and

when $4<x$
or sketch!
e) $\frac{4 x+9}{4 x-1} \leq \frac{x+5}{x}$

$$
\begin{aligned}
& \frac{4 x+9}{4 x-1}-\frac{x+5}{x} \leq 0 \\
& \frac{x(4 x+9)-(4 x-1)(x+5)}{x(4 x-1)} \leq 0 \\
& \frac{4 x^{2}+9 x-4 x^{2}-19 x+5}{x(4 x-1)} \leq 0 \\
& \frac{-10 x+5}{x(4 x-1)} \leq 0 \\
& \frac{-5(2 x-1)}{x(4 x-1)} \leq 0
\end{aligned}
$$

$$
\begin{aligned}
& \text { zeros: } \frac{1}{2} \\
& \text { VA: } x=0 \quad x=\frac{1}{4}
\end{aligned}
$$



or factors |  | $x<0$ | $0<x<\frac{1}{4}$ | $\frac{1}{4}<x<\frac{1}{2}$ | $\frac{1}{2}<x$ |
| :---: | :---: | :---: | :---: | :---: |
| $\frac{-5}{2}$ | - | - | - | - |
| $2 x-1$ | - | - | - | + |
| $x$ | - | + | + | + |
| $4 x-1$ | - | - | + | + |
| $\begin{array}{l}\text { sign of } \\ \text { frit }\end{array}$ | + | - | + | - |

$$
\begin{gathered}
\text { when } 0<x<\frac{1}{4} \\
\text { or } \frac{1}{2} \leq x
\end{gathered}
$$

20

$$
\begin{aligned}
\cos 2 \theta & =-0.9541 & \text { Period }=\frac{2 \pi}{2} \\
\text { Let } x & =2 \theta & =\pi \\
\cos x & =-0.9541 & \\
\cos x_{r} & =0.9541 & \\
x_{r} & =0.3042 &
\end{aligned}
$$

$x_{1}=\pi-x_{r}$

$$
\begin{array}{rlrl}
x_{1} \doteq 2.8374 \rightarrow \theta_{1} & =x_{1} \div 2 \\
\theta_{1} & =1.4187 & \theta_{3}=\theta_{1}+\text { period } \\
\theta_{3} \doteq 4.5603
\end{array}
$$

$x_{2}=\pi+x_{r}$

$$
\begin{aligned}
x_{2} \doteq 3.4458 \rightarrow \theta_{2} & =x_{2} \div 2 \\
\theta_{2} & \doteq 1.7229
\end{aligned} \quad \begin{aligned}
& \theta_{4}=\theta_{2}+\text { peril } \\
& \theta_{4} \doteq 4.8645
\end{aligned}
$$

$$
\text { for more answers add/subtract } n \pi
$$

$$
n \in \mathbb{Z} .
$$

g) $\sin \theta-\sin \theta \tan \theta=0$

$$
\sin \theta-\sin \theta \tan \theta=0
$$

$\sin \theta(1-\tan \theta)=0$
$\sin \theta=0$ or $1-\tan \theta=0$

$$
1=\tan \theta
$$

$$
\theta=0, \pi, 2 \pi \quad \begin{array}{ll}
\frac{\pi}{4} & =\theta_{4} \\
\theta_{5} & =\frac{\pi}{4}+\pi \\
\theta_{5} & =\frac{5 \pi}{4}
\end{array}
$$

$$
\theta=0, \frac{\pi}{4}, \pi, \frac{5 \pi}{4}, 2 \pi
$$

h) $6 \sin ^{2} \theta-5 \cos \theta-2=0$
) $6 \sin ^{2} \theta-5 \cos \theta-2=0$

$$
6\left(1-\cos ^{2} \theta\right)-5 \cos \theta-2=0
$$

$$
6-6 \cos ^{2} \theta-5 \cos \theta-2=0
$$

$$
-6 \cos ^{2} \theta-5 \cos \theta+4=0
$$

$$
6 \cos ^{2} \theta+5 \cos \theta-4=0
$$

$$
p=-24
$$

$$
6 \cos ^{2} \theta+5 \cos \theta-4=0 \quad s=5
$$

$$
(2 \cos \theta-1)(3 \cos \theta+4)=0
$$

$$
\cos \theta=\frac{1}{2} \quad \cos \theta=\frac{-4}{3}
$$

$$
\begin{aligned}
& \frac{8}{6}, \frac{-3}{6} \\
& \frac{4}{3}, \frac{-1}{2}
\end{aligned}
$$

$$
\theta_{1}=\frac{\pi}{3} \quad \cos \theta_{R}=\frac{+4}{3}
$$

$$
\theta_{2}=2 \pi-\theta_{1} \quad \text { no sol } 1=\frac{4}{3}>1
$$

$$
\theta_{2}=\frac{5 \pi}{3}
$$

$$
\theta=\frac{\pi}{3}, \quad \frac{5 \pi}{3}
$$

18) The graph of the function $p(x)=3^{x} \sin x$ is shown on the right. Use the graph to estimate the answer to the following questions then verify your answers) using the equation.
a) evaluate $p(1)$
a) $p(1)$ find $y$ value when $x=1$

$$
\begin{aligned}
& p(1) \doteq 2.6 \\
& \text { verify: } p(1) \\
&=3^{\prime} \sin 1 \\
& p(1) \doteq 3(0.8414 \ldots) \\
& p(1) \doteq 2.52
\end{aligned}
$$


b) solve for $a$ if $p(a)=8$

$$
\begin{array}{ll}
p(a)=8 & \text { find } x \text { when } y=8 \\
a \doteq 1.99 & a \doteq 2.7
\end{array}
$$

c) the interval for which $y \leq 2$

$$
\begin{aligned}
& y \leq 2 \\
& \text { find when } y=2 \\
& \qquad \begin{array}{l}
x \doteq 0.85 \quad x \doteq 3.07 \\
\text { verify } p(0.85) \doteq 1.91 \quad p(3.07)=2.09 \\
\quad 0 . \\
y \leq 2 \text { when } \quad x<0.85 \\
\text { or } x>3.07
\end{array}
\end{aligned}
$$

19) For the function defined by $f(x)=k(x+1)^{2}(x-2)(x-4)$
a) Determine the value of $k$, if $(1,-24)$ is a point on the graph of the function

$$
\text { a) } \begin{aligned}
-24 & =k(2)^{2}(-1)(-3) \\
-24 & =12 K \\
-2 & =K \\
f(x) & =-2(x+1)^{2}(x-2)(x-4)
\end{aligned}
$$

b) solve for p if $(3, \mathrm{p})$ is a point on the graph of the function

$$
\begin{aligned}
& p=f(3) \\
& p=-2(4)^{2}(1)(-1) \\
& p=32
\end{aligned}
$$

c) considering the end behaviours and the zeros, state where $f(x)>0$
$\begin{aligned} & \text {-) zeros: }-1 \text { DR } \therefore \text { sign does not change } \\ & 2,4 \text { SR } \therefore \text { sign changes }\end{aligned}$
2,4 SR $\therefore$ sign changes
ceff. -ve $\therefore$ ends -ve

$f(x)>0$ when $2<x<4$

## Combination of functions:

20) Determine $h(x)=(f \circ g)(x)$ when $f(x)=2 x^{4}-3 x^{2}$ and $g(x)=\sqrt{x-3}$ and state the domain and range of $h(x)$.
a) $D_{n} \in D_{g}$ when $g(x) \in D_{f}$

$$
\begin{aligned}
& D_{g}: x \geqslant 3 \text { and all } g(x) \in D_{f} \\
& D_{n}:\{x \mid x \geqslant 3, x \in \mathbb{R}\}
\end{aligned}
$$

b) $R_{n} \in R_{f}$ when $g(x) \in D_{f}$

$$
\begin{gathered}
R_{f}: y \geqslant-1.125 \text { and all } g(x) \in D_{f} \\
R_{h}:\{y \mid y \geqslant-1.125, y \in R\} \\
\quad \begin{array}{r}
\uparrow \\
\\
\quad \text { min. vale of } f(x)
\end{array}
\end{gathered}
$$

21) Given $D_{f}=\{x \mid-5 \leq x \leq 8, \quad x \in \mathfrak{R}\}$ and $D_{g}=\{x \mid-12 \leq x \leq 3, \quad x \in \mathfrak{R}\}$ determine
a) $D_{f+g}$
b) $D_{g \cdot f}$
a) $D_{f+g}$ : "overlap"

$$
D_{f+g}=\{x \mid-5 \leqslant x \leqslant 3, x \in \mathbb{R}\}
$$

b) Dg.f: "overlap"

$$
D_{g \cdot f}=\{x \mid-5 \leq x \leq 3, x \in \mathbb{R}\}
$$

22) The Graphs of $y=f(x)$ and $y=g(x)$ are given below.


On the same grid sketch
a) $y=f(x)+g(x)$
b) $y=f(x) \cdot g(x)$
23) Given $s(x)=\sin x+2 \cos x$,
a) determine $D_{s} \quad$ b) at most, what is the range of the function?
a) $D_{s}=\{x \mid x \in \mathbb{R}\}$

$$
\begin{aligned}
& \text { (both fuck have the } \\
& \text { same domain, } x \in \mathbb{R} \text { ) }
\end{aligned}
$$

b) at most $R_{S}=R_{\sin x}+R_{2 \cos x}$

$$
\begin{aligned}
& \text { but since phase shift, never equal to the actual sum! } \\
& \text { Rs: }\{y \mid-3<y<3, y \in \mathbb{R}\}
\end{aligned}
$$

24) Given $d(x)=\tan x+\log x$, determine $D_{d}$
$D_{d}:\left\{x \mid x>0, \quad x \neq \frac{\pi}{2}+n \pi, n \in N, x \in \mathbb{R}\right\}$

$$
\begin{gathered}
\uparrow \\
\text { for } \log x
\end{gathered} \begin{gathered}
\uparrow \\
\text { restrictions on } \tan x
\end{gathered}
$$

25) What is the maximum number of zeros possible for $p(x)=(x+2)(x+3)(x-4) \log x$ ? Do you think there will actually be that many zeros? Justify your answers.

$$
\begin{aligned}
& p(x)=(x+2)(x+3)(x-4) \log x \\
& p(x)=\text { cubic. logarithm } \\
& \text { The fact could have } 4 \text { zeros (3 from cubic } \\
& \text { an } 1 \text { from log) }
\end{aligned}
$$

This will not be the actual $\#$ of zeros
since two of the zeros of the cubic
are regative and the domain - because
of $\log x-$ is restricted to $x>0$.
$\underset{\text { Zeros }}{\text { actual }}=1,4$
26) Given $f(x)=\frac{x}{x+1}$ and $g(x)=\cos x$, determine (Keep your answers within $[0,2 \pi]$.)
a) $D_{f \cdot g}$
b) $D_{f \div g}$
c) zeros, holes and vertical asymptotes of $g \div f$
la) $D_{f . g}$ : overlap of $D_{f}$ and $D_{g}$
$D_{f \cdot g}=\{x \mid 0 \leq x \leq 2 \pi, x \in \pi\}$

$$
x \neq-1 \text { not seeded } \because \text { not in domain }
$$

b) $D_{f i g}$ : overlap of $D_{f}$ and $D_{g}$ additional $2:$

$$
\begin{aligned}
D_{f \div g}= & \left\{x \left\lvert\, x \neq \frac{\pi}{2}\right., \frac{3 \pi}{2}, \quad 0 \leq x \leq 2 \pi, x \in \mathbb{R}\right\} \\
& \text { restriction } x \neq+ \text { not raced } \because \text { not in } D_{g}
\end{aligned}
$$

c) zeros: $\frac{\pi}{2}, \frac{3 \pi}{2}$; hole: none since $\mathrm{x}=-1$ is not in the domain; VA $\mathrm{x}=0$

## Word Problems:

27) Distance in kilometres above sea level is given by the formula $d=\frac{500(\log P-2)}{27}$, where $P$ is the atmospheric pressure measured in kiloPascals, kPa .
a) At the top of the highest mountain in Shelbyville, the atmospheric pressure was recorded as being 220 kPa . Calculate the height of the mountain above sea level.

$$
P=220
$$

$d=\frac{500(\log 220-2)}{27}$
$d=6.34$
The highest mountain in Shelby ville is approx.
6.34 km above sea level.
b) The town of Springfield has a mountain with a peak 4.5 km above sea level. Calculate the atmospheric pressure at the top of the mountain.

$$
\begin{aligned}
& 4.5=\frac{500(\log P-2)}{27} \\
& 121.5=500(\log P-2) \\
& 0.243=\log P-2 \\
& 2.243=\log P \\
& P=10^{2.243} \\
& \Leftrightarrow \quad \equiv 174.98 \\
& \text { The at mospheric pressure at the top of the } \\
& \text { mountain is approx. } 175 \mathrm{kPa} .
\end{aligned}
$$

c) In the year 1980, both towns had an earthquake. Springfield's earthquake measured 7.5 on the Richter Scale. The magnitude of the earthquake was $10^{7.5}$. The earthquake in Shelbyville measured 6.4. How many times more intense was Springfield's earthquake when compared to Shelbyville's earthquake. Recall: $M=\log \left(\frac{I}{I_{o}}\right)$

$$
\begin{aligned}
& \text { c) } M=\log \frac{I}{I_{0}} \\
& \Delta M=\log \frac{I_{\text {springier }}}{I \text { shalognile }} \\
& \text { 7.5-6.4 }=\log \frac{I_{\text {spar }}}{I_{\text {sulbog }}} \\
& \Leftrightarrow \frac{I_{s_{p o x y}}}{I_{\text {gL }}}=10^{1.1} \\
& \frac{I_{\text {spray }}}{I_{\text {slit }}} \stackrel{12.59}{ } \\
& \begin{array}{l}
\text { Springfield's earthquake is approx. } 12.59 \text { times } \\
\text { more inverse than shelbyville's earthquake. }
\end{array}
\end{aligned}
$$

28) The volume of air in the lungs during normal breathing can be modeled by a sinusoidal function of time. Suppose a person's lungs contain from 2200 mL to 2800 mL of air during normal breathing. Suppose a normal breath takes 4 seconds, and that $t=0 \mathrm{~s}$ corresponds to a minimum volume.
a) Let $V$ represent the volume of air in a person's lungs. Draw a graph of Volume versus time for 20 seconds.

b) State the period, amplitude, phase shift and vertical translation for the function.

$$
\begin{aligned}
\text { Period }=4 & \text { Amplitude } & =\frac{\left|M_{a x}-M_{i n}\right|}{2} & \text { P.S: } 0 \text { for }-\operatorname{cosine} \\
K=\frac{2 \pi}{4} & & =300 & \text { V. translation }
\end{aligned}=\frac{M_{a x}+M_{n}}{2} .
$$

c) Write a possible equation for the volume of air as a function of time.

$$
\begin{aligned}
& V(t)=-300 \cos \frac{\pi}{2} t+2500 \\
& \text { where trep. The time in seconds and } V(t) \text { is } \\
& \text { The volume in } \mathrm{mL} \text {. }
\end{aligned}
$$

d) Describe how the graph would change if the person breaths more rapidly.

$$
\begin{aligned}
& \text { It a person breaths more rapidly } \\
& \text { the period will be shorter } \therefore K \text { wald be larger. }
\end{aligned}
$$

e) Describe how the graph would change if the person took bigger breaths.

If a person took deeper breaths
the amplitude would be greater
since they would breath in more air.
(The minimum amount would probably
stay the same $\therefore$ graph would also shift p).
f) Determine the amount of air in the lungs after 8 seconds.

$$
\begin{aligned}
& V(8)=-300 \cos \frac{\pi}{2} \cdot 8+2500 \\
& V(8)=-300 \cos 4 \pi+2500 \\
& V(8)=2200 \quad \text { (matches graph) } \\
& \text { After } 8 \text { seconds the is } 2200 \mathrm{~mL} \text { of air in the lungs. }
\end{aligned}
$$

g) Determine when, within the first 8 seconds, the volume is 2400 mL .

$$
\begin{aligned}
& \frac{1}{3}=\cos \theta \\
& V(t)=2400 \\
& 2400=-300 \cos \frac{\pi}{2} t+2500 \\
& 2400-2500=-300 \cos \frac{\pi}{2} t \\
& -100=-300 \cos \frac{\pi}{2} t \\
& \frac{1}{3}=\cos \frac{\pi}{2} t \\
& \text { Let } \theta=\frac{\pi}{2} t \\
& \begin{aligned}
1.2310 \div \theta, \quad \rightarrow \quad t_{1} & =\theta_{1} \div \frac{\pi}{2} \\
t_{1} & =0.7837 \quad t_{3}=t_{1} \text { +period }
\end{aligned} \\
& \theta_{2}=2 \pi-\theta_{1} \quad t_{3}=4.7837 \\
& \theta_{2} \doteq 5.0522 \rightarrow t_{2} \doteq 3.2163 \quad t_{4}=t_{2}+\text { period } \\
& t_{4}=7.2163 \\
& t_{5}=t_{3}+\text { period } \\
& >8 \text { second } \therefore \text { inadmissable } \\
& \text { The volume is } 2400 \mathrm{~mL} \text { when } t \neq 0.78 \mathrm{sec} \text {, } \\
& 3.22 \mathrm{sec}, 4.78 \mathrm{sec} \text {, and } 7.22 \mathrm{sec} \text {. }
\end{aligned}
$$

29) You have just walked out the front door of your home. You notice that it closes quickly at first and then closes more slowly. In fact, a model of the movement of a closing door is given by $d(t)=200 t(2)^{-t}$, where $d$ represents the width of the opening in $\mathrm{cm} t$ seconds after opening the door.
a) determine the width of the opening after $2 \mathrm{sec} ., 4 \mathrm{sec} ., 6 \mathrm{sec}, 10 \mathrm{sec}$.
a) $d(2)=200(2)(2)^{-2}$
$d(6)=200(6)(2)^{-6}$
$\alpha(2)=100$
$d(6)=18.75$
$d(4)=200(4)(2)^{-4}$
$d(10)=200(10)(2)^{-10}$
$d(4)=50$
$d(10) \doteq 1.95$
The door opening is 100 cm after $2 \mathrm{sec}, 50 \mathrm{~cm}$
after $4 \mathrm{sec}, 18.75 \mathrm{cmafte} 6 \mathrm{sec}$ and 1.95 cm after 10 sec .
b) Determine the average rate of change from $t=0 \mathrm{sec}$. to $\mathrm{t}=1.5 \mathrm{sec}$.. What does this tell you about the movement of the door.

$$
\begin{array}{ll}
\text { avg. } R_{0} C=\frac{d(1.5)-d(0)}{1.5-0} \\
\text { avg. } R_{0} C & =\frac{106.07-0}{1.5} \\
\text { avg. } R_{0} C=70.71 & \text { The door is } \\
& \begin{array}{ll}
\text { opening at a rate } \\
\text { of } 70.71 \mathrm{~cm} / \mathrm{sec} .
\end{array}
\end{array}
$$

c) Determine the average rate of change from $t=3 \mathrm{sec}$. to $t=6 \mathrm{sec}$.. What does this tell you about the movement of the door.
avg. $R_{0} C=\frac{d(6)-d(3)}{6-3}$
avg. $R_{0} C=\frac{18.75-75}{3}$
avg. $\operatorname{RO} C=-18.75$
The door is closing at a rate of $18.75 \mathrm{~cm} / \mathrm{sec}$.
d) sketch a graph to model the movement of the door. How does your sketch support the conclusions you reached in b) and c)?


## Identities:

30) Prove the following identities.
a) $\frac{\cos \theta}{1+\sin \theta}+\frac{\cos \theta}{1-\sin \theta}=\frac{2}{\cos \theta}$
b) $\frac{1}{\sec \theta+\tan \theta}=\sec \theta-\tan \theta$
a) $\frac{\cos \theta}{1+\sin \theta}+\frac{\cos \theta}{1-\sin \theta}=\frac{2}{\cos \theta}$
LS: $\frac{\cos \theta}{1+\sin \theta}+\frac{\cos \theta}{1-\sin \theta}$
$=\frac{\cos \theta(1-\sin \theta)+\cos \theta(1+\sin \theta)}{(1+\sin \theta)(1-\sin \theta)}$
$\cos \theta-\sin \theta \cos \theta+\cos \theta+\sin \theta \cos \theta$
$=\frac{2 \cos \theta}{\cos ^{x} \theta}$
$=\frac{2}{\cos \theta}$
$=R S$ QED

$$
\begin{aligned}
& L S: \frac{1}{\sec \theta+\tan \theta} \\
&= 1 \div(\sec \theta+\tan \theta) \\
&=1 \div\left(\frac{1}{\cos \theta}+\frac{\sin \theta}{\cos \theta}\right) \\
&= 1 \div \frac{1+\sin \theta}{\cos \theta} \\
&= 1 \cdot \frac{\cos \theta}{1+\sin \theta} \\
&= \frac{\cos \theta}{1+\sin \theta} \cdot \frac{1-\sin \theta}{1-\sin \theta} \\
&=\frac{\cos \theta(1-\sin \theta)}{1-\sin 2} \\
&=\frac{\cos \theta(1-\sin \theta)}{\cos { }^{2} \theta} \\
&=\frac{1-\sin \theta}{\cos \theta} \\
&=\frac{1}{\cos \theta}-\frac{\sin \theta}{\cos \theta} \\
&=\sec \theta-\tan \theta \\
&=R S \\
& Q E D
\end{aligned}
$$

c) $\frac{1}{1+\sin \theta}=\sec ^{2} \theta-\frac{\tan \theta}{\cos \theta}$
RS: $\sec ^{2} \theta-\frac{\tan \theta}{\cos \theta}$
d) $(1-\cos \beta)^{2}+\sin ^{2} \beta=2(1-\cos \beta)$ LS: $(1-\cos B)^{2}+\sin ^{2} \beta$
$=1-2 \cos B+\cos ^{2} B+\sin ^{2} B$
$=\frac{1}{\cos ^{2} \theta}-\tan \theta \cdot \frac{1}{\cos \theta}$

$$
=1-2 \cos 13+1
$$

$$
=2-2 \cos \beta
$$

$=\frac{1}{\cos ^{2} \theta}-\frac{\sin \theta}{\cos ^{2} \theta}$
$=2(1-\cos B)$
$=R S$ QED
$=\frac{1-\sin \theta}{\cos ^{2} \theta}$
$=\frac{1-\sin \theta}{1-\sin ^{2} \theta}$
$=\frac{1-\sin \theta}{(1+\sin \theta)(1-\sin \theta)}$
$=\frac{1}{1+\sin \theta}$
$=25$ QED

## Miscellaneous:

31) $2 x^{3}+3 x^{2}+k x-5$ is divided by $x+2$ to give a remainder of 2 . Determine $k$.
let $P(x)=2 x^{3}+3 x^{2}+K x-5$

$$
\begin{aligned}
\text { remainder thm: } \quad P(-2) & =2 \\
2(-2)^{3}+3(-2)^{2}+k(-2)-5 & =2 \\
-16+12-2 k-5 & =2 \\
-2 k & =11 \\
k & =\frac{-11}{2}
\end{aligned}
$$

32) State the quotient and remainder when $2 x^{3}+5 x^{2}-x-5$ is divided by $x+2$.

$$
\begin{aligned}
\begin{array}{l}
\frac{-\left(2 x^{3}+4 x^{2}\right) \mid}{x^{2}-x-5} \\
\frac{-\left(x^{2}+2 x\right)}{2 x^{3}+5} \\
\left.\frac{-(-3 x-5}{1}-6\right) \\
1
\end{array} & \text { quotient: } 2 x^{2}+x-3
\end{aligned}
$$

33) Use the Factor Theorem to fully factor: $x^{3}-4 x^{2}-11 x+30$

$$
\begin{gathered}
\text { Let } \begin{array}{c}
P(x)=x^{3}-4 x^{2}-11 x+30 \\
P(2)=8-16-22+30 \\
P(2)=0 \therefore x-2 \text { is a factor } \\
2 \frac{11-4-11-30}{\downarrow} \frac{2}{-2} \frac{-4}{-15} \frac{-30}{0} \\
x^{3}-4 x^{2}-11 x+30=(x-2)\left(x^{2}-2 x-15\right) \\
x^{3}-4 x^{2}-11 x+30=(x-2)(x-3)(x+5)
\end{array}
\end{gathered}
$$

34) Convert the following radians to degrees. Round your answer to one decimal place, if necessary.
a) $\frac{5 \pi}{6}$
b) $\frac{-3 \pi}{8}$
c) 2.678
35) a) $\frac{5 \pi}{6}=\frac{5}{6} \pi \cdot \frac{180}{\pi}$
b) $\frac{-3 \pi}{8}=\frac{-3}{8} \pi \cdot \frac{180}{7}$
$2.678=2.678 \cdot \frac{180}{\pi}$
$=150^{\circ}$

$$
=-67.5^{0}
$$

$$
\pm 153.4^{\circ}
$$

35) The point $\mathrm{P}(-5,4)$ is on the terminal arm of an angle of measure $\theta$ in standard position.
a) Sketch the principal angle.
b) Determine the exact value of $\sin \theta$.
a)

b) $x^{2}+y^{2}=r^{2}$ $25+16=r^{2}$
$41=r^{2}$ $\sin \theta=\frac{4}{\sqrt{41}}$
c) Determine the exact value of $\cos \left(\theta-\frac{\pi}{6}\right)$

$$
\text { c) } \begin{aligned}
& \cos \left(\theta-\frac{\pi}{6}\right) \\
= & \cos \theta \cos \frac{\pi}{6}+\sin \theta \sin \frac{\pi}{6} \\
= & \frac{-5}{\sqrt{41}} \cdot \frac{\sqrt{3}}{2}+\frac{4}{\sqrt{41}} \cdot \frac{1}{2} \\
= & \frac{-5 \sqrt{3}+4}{2 \sqrt{41}}
\end{aligned}
$$

d) Determine the value of $\theta$, to the nearest degree, where $0 \leq \theta \leq 2 \pi$.

$$
\begin{aligned}
& \theta=\sin ^{-1}\left(\frac{4}{\sqrt{47}}\right) \\
& \theta=0.6747
\end{aligned}
$$

36) Determine the smallest positive co-terminal angle to $\frac{13 \pi}{5}$. Determine a co-terminal angle that is larger than $\frac{13 \pi}{5}$. Determine a negative co-terminal angle.

$$
\begin{aligned}
\text { tee co-terminal } & =\frac{13 \pi}{5}-2 \pi & & \text {-ve coterminal } \\
& =\frac{3 \pi}{5} & & =\frac{3 \pi}{5}-2 \pi \\
\text { larger co-terminal } & =\frac{13 \pi}{5}+2 \pi & & =-\frac{7 \pi}{5} \\
& =\frac{23 \pi}{5} & &
\end{aligned}
$$

37) Determine the exact value for each of the following:
a) $\sin \frac{5 \pi}{4}$
a) $\sin \frac{5 \pi}{4}$


$$
=-\sin \frac{\pi}{4}
$$

$$
=-\frac{\sqrt{2}}{2}
$$

b) $\cos \frac{11 \pi}{6}$
b) $\cos \frac{11 \pi}{6}$

$=\cos \frac{\pi}{6}$
$=\frac{\sqrt{3}}{2}$
c) $\tan \frac{\pi}{8}$

$$
\begin{aligned}
\tan \frac{\pi}{4} & =\frac{2 \tan \frac{\pi}{8}}{1-\tan ^{2} \frac{\pi}{8}} \\
1 & =\frac{2 \tan \frac{\pi}{8}}{1-\tan ^{2} \frac{\pi}{8}} \\
1-\tan ^{2} \frac{\pi}{8} & =2 \tan \frac{\pi}{8}
\end{aligned}
$$

$$
0=\tan ^{2} \frac{\pi}{8}+2 \tan \frac{\pi}{6}-1
$$

$$
\tan \frac{\pi}{8}=\frac{-2 \pm \sqrt{4+4}}{2}
$$

$$
\tan \frac{\pi}{8}=\frac{-2 \pm \sqrt{8}}{2}
$$

$$
\tan \frac{\pi}{8}=\frac{-2 \pm 2 \sqrt{2}}{2}
$$

$$
\tan \frac{\pi}{8}=\frac{-2 \pm \sqrt{2}}{2}
$$

d) $\csc \frac{7 \pi}{12}$

$$
\begin{aligned}
& \text { d) } \begin{aligned}
& \csc \frac{7 \pi}{12}=\text { reciprocal of } \sin \frac{7 \pi}{12} \\
& \sin \frac{7 \pi}{12}=\sin \left(\frac{3 \pi}{12}+\frac{4 \pi}{12}\right) \\
& \sin \frac{7 \pi}{12}=\sin \left(\frac{\pi}{4}+\frac{\pi}{3}\right) \\
& \sin \frac{7 \pi}{12}=\sin \frac{\pi}{4} \cdot \cos \frac{\pi}{3}+\cos \frac{\pi}{4} \cdot \sin \frac{\pi}{3} \\
& \sin \frac{7 \pi}{12}=\frac{\sqrt{2}}{2} \cdot \frac{1}{2}+\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} \\
& \sin \frac{7 \pi}{12}=\frac{\sqrt{2}+\sqrt{6}}{4} \\
& \csc \frac{7 \pi}{12}=\frac{4}{\sqrt{2}+\sqrt{6}}
\end{aligned} .
\end{aligned}
$$

38) Express each of the following as a co-function
a) $\sin \frac{27 \pi}{22}$
b) $\sec \frac{17 \pi}{30}$
a) $\sin \frac{27 \pi}{22}$

b) $\sec \frac{17 \pi}{30}$
$=\cos \frac{6 \pi}{22}$
$=\cos \frac{3 \pi}{11}$

$$
\begin{aligned}
& =-\csc \frac{2 \pi}{30} \\
& =-\csc \frac{\pi}{15}
\end{aligned}
$$


39) Express $\log _{11} 3+2 \log _{11} 5-\log _{11} 7$ as a single logarithm.

$$
\begin{aligned}
& \log _{11} 3+2 \log _{11} 5-\log _{11} 7 \\
= & \log _{11} 3+\log _{11} 25-\log _{11} 7 \\
= & \log _{11} \frac{75}{7}
\end{aligned}
$$

40) Express $\log _{4} 7$ as a single logarithm with base 2 .

$$
\begin{aligned}
& \log _{4} 7=\frac{\log _{2} 7}{\log _{2} 4} \\
& \log _{4} 7=\frac{\log _{2} 7}{2} \\
& \log _{7} 7=\frac{1}{2} \log _{2} 7 \\
& \log _{4} 7=\log _{2} \sqrt{7}
\end{aligned}
$$

