

Identities are equations that yield a true statement no matter what value the variable takes on.

e.g. $\sin \theta = \sin(\theta + 2\pi)$
for any θ .

To show an equation is not an identity, you only need a single counterexample.
→ one value that does not work.

Identities

Reciprocal: $\csc \theta = \frac{1}{\sin \theta}$ $\sec \theta = \frac{1}{\cos \theta}$
 $\cot \theta = \frac{1}{\tan \theta}$

Quotient: $\tan \theta = \frac{\sin \theta}{\cos \theta}$ $\cot \theta = \frac{\cos \theta}{\sin \theta}$

Pythagorean: (see notes)

Addition Formulas: (" ")

Subtraction " : (" ")

Double Angle Formulas (see notes)

Strategies:

- only simplify one side of the identity at a time
- Start with the more complicated side and make it simpler (if possible)
- NEVER move anything from one side to another.
- express $\csc \theta$, $\sec \theta$, $\tan \theta$, & $\cot \theta$ in terms of $\sin \theta$, $\cos \theta$
- Simplify algebraically, which often means writing one side with a common denominator.
- look for opportunities to factor.

Ex. 1. Prove $\frac{1}{\sec x} + \frac{\sin x}{\cot x} = \sec x$

$$\begin{aligned} LS &= \frac{1}{\sec x} + \frac{\sin x}{\cot x} \\ &= \cos x + \frac{\sin x}{\frac{\cos x}{\sin x}} \times \frac{\sin x}{\cos x} \\ &= \cos x + \frac{\sin^2 x}{\cos x} \\ &= \frac{\cos^2 x + \sin^2 x}{\cos x} \\ &= \frac{1}{\cos x} \quad RS = \sec x \end{aligned}$$

$$LS = \sec x \quad \therefore LS = RS \\ \therefore \text{identity is true}$$

Ex.2 Prove

$$\cos(x+y)\cos(x-y) = \cos^2 x + \cos^2 y - 1$$

$$LS = \cos(x+y)\cos(x-y)$$

$$= [\cos x \cos y - \sin x \sin y][\cos x \cos y + \sin x \sin y]$$

$(a-b)(a+b) = a^2 - b^2$

$$= \cos^2 x \cos^2 y - \sin^2 x \sin^2 y$$

want cos terms, not sin terms

$$= \cos^2 x \cos^2 y - (1 - \cos^2 x)(1 - \cos^2 y)$$

$$= \cos^2 x \cos^2 y - [1 - \cos^2 x - \cos^2 y + \cos^2 x \cos^2 y]$$

$$= \cancel{\cos^2 x \cos^2 y} - 1 + \cos^2 x + \cos^2 y - \cancel{\cos^2 x \cos^2 y}$$

$$= \cos^2 x + \cos^2 y - 1$$

$$\therefore LS = RS$$

∴ identity is true

H/W: p.417 #5b, 6, 7, 9, 10abcd

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