

Vertex Form by Completing the Square

May 4/2010

How can we find the vertex of a quadratic relation in standard form,  $y = ax^2 + bx + c$ ?

1. Factored Form  $\longrightarrow$  Find zeroes  
 $\longrightarrow$  Symmetry to find  $x_v$
2. Partial Factoring  $\longrightarrow$  sub y-intercept  
 $\longrightarrow$  find matching point  
 $\longrightarrow$  symmetry to find  $x_v$

or,

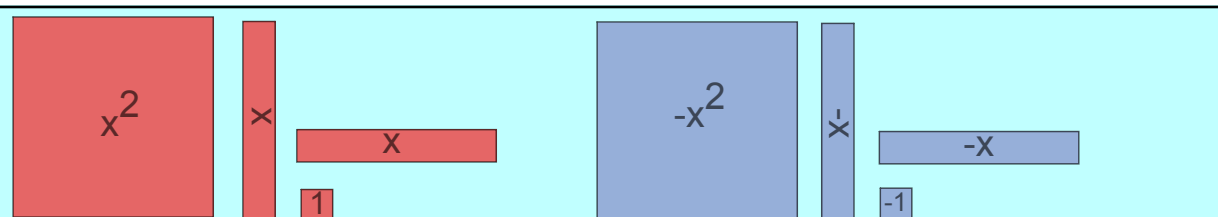
3. **Complete the Square  $\longrightarrow$  Vertex Form**

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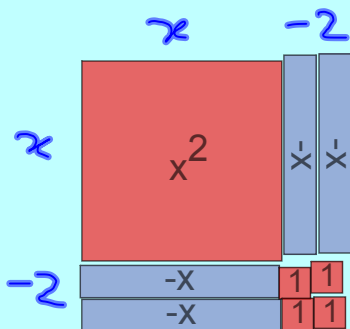
$x^2 + 6x + \underline{9} = (x+3)^2$

Diagram illustrating the process of completing the square for a quadratic equation. The diagram shows the decomposition of  $x^2 + 6x + 9$  into  $(x+3)^2$  using algebra tiles. The top row shows the components: a large red square for  $x^2$ , a vertical red strip for  $6x$  (labeled with a red  $x$ ), and a small red square for  $9$  (labeled with a red  $1$ ). To the right, a blue square represents  $-x^2$ , a blue vertical strip for  $-6x$  (labeled with a blue  $x$ ), and a small blue square for  $-9$  (labeled with a blue  $-1$ ). Below this, the equation  $x^2 + 6x + 9 = (x+3)^2$  is written with blue annotations. The bottom part shows the completed square as a large red square ( $x^2$ ) with a vertical red strip ( $6x$ ) and a small red square ( $9$ ) attached to its right side. The total width is labeled  $x+3$  and the total height is labeled  $x+3$ . The vertical strip is divided into three  $2 \times 1$  rectangles, each labeled  $x$  and  $1$ . The small square is divided into three  $1 \times 1$  squares, each labeled  $1$ .

Mar 25-8:02 AM



$$x^2 - 4x + \underline{4} = (x - 2)^2$$



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Recall vertex form:  $y = a(x - h)^2 + k$

Note that  $(x - h)^2$  is a perfect square. What is missing from these perfect squares?

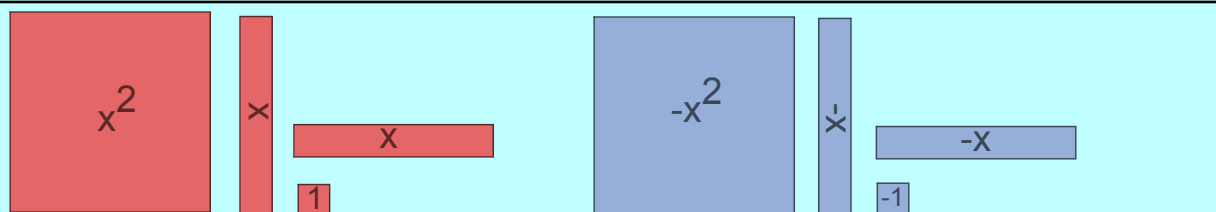
$$(a) x^2 + 10x + \underline{25} = (x + 5)^2$$

$$\frac{10}{2} = 5 \quad 5^2 = 25$$

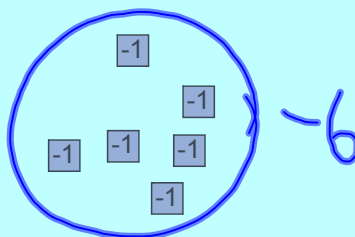
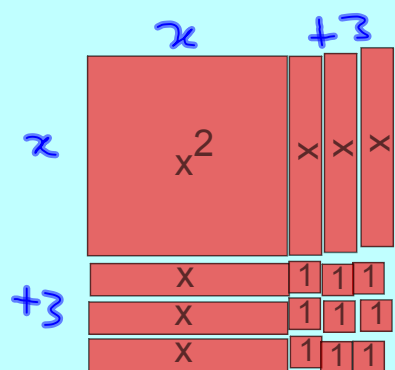
$$(b) x^2 - 18x + \underline{81} = (x - 9)^2$$

$$-\frac{18}{2} = -9 \quad (-9)^2 = 81$$

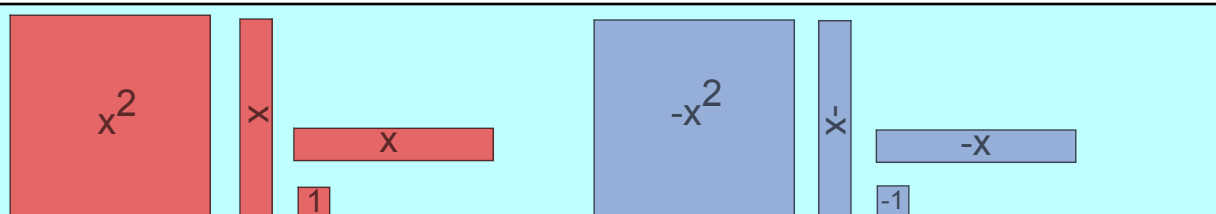
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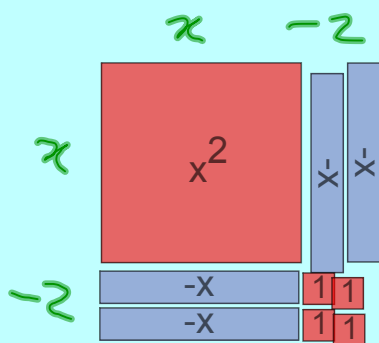
$$x^2 + 6x + 3 = (x+3)^2 - 6$$



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$$x^2 - 4x - 3 = (x-2)^2 - 7$$



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To go from standard form to vertex form, we force a perfect square into our equation.

(c)  $y = x^2 + 12x - 7$

*make a perfect square*

$$y = (x^2 + 12x + 36) - 36 - 7$$

*zero*

*perfect square*

$$y = (x + 6)^2 - 43$$

	$x + 6$	
	$x^2$	$6x$
$x$		
$+$		
$6$	$6x$	$36$

$-36 - 7 = -43$

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(d)  $y = x^2 - 20x + 15$

$$y = (x - 10)^2 - 85$$

	$x$	$-10$
	$x^2$	$-10x$
$x$		
$-10$	$-10x$	$+100$

$-100 + 15$

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If the value in front of  $x^2$  is not 1 (the 'a' term), we must factor that number out of all x terms.

(e)  $y = 3x^2 + 12x + 11$

$$y = 3(x^2 + 4x) + 11$$

$$y = 3(\underbrace{x^2 + 4x + 4}_{(x+2)^2} - 4) + 11$$

$$y = 3[(x+2)^2 - 4] + 11$$

$$y = 3(x+2)^2 - 12 + 11$$

$$y = 3(x+2)^2 - 1$$

(f)  $y = -x^2 + 6x + 13$

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Assigned Work:

p. 390 # 1, 2, 4, 9, 10

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