Quadratic Relations in Factored Form   *Apr. 19/2010*

The equation of a quadratic relation may be written
in several forms:

    1. factored form: $y = a(x - s)(x - t)$ $a \neq 0$

    2. standard form: $y = ax^2 + bx + c$
    3. vertex form: $y = a(x - h)^2 + k$

In the factored form,

    a tells us the sign of $\Delta^2 y$
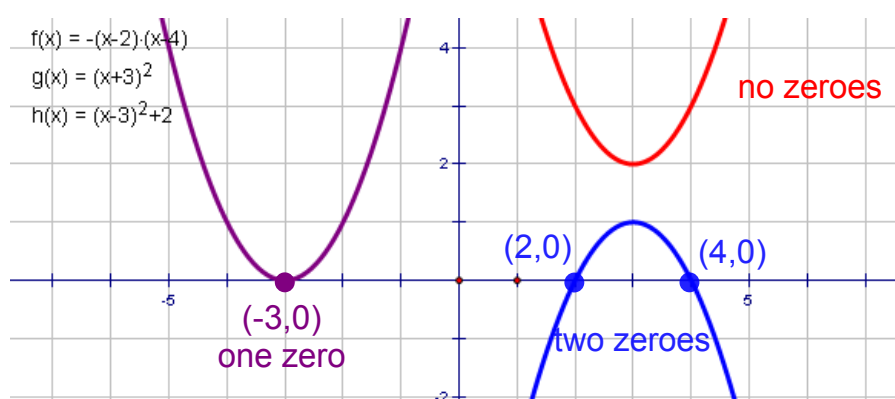
    s and t are the zeroes, or x-intercepts

> Recall:
> $\Delta^2 y > 0$, opens up
> $\Delta^2 y < 0$, opens down

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Depending upon the location of the <u>vertex</u>, and
whether the parabola opens up or down, it may have
0, 1, or 2 <u>distinct</u> (unique) zeroes.

$f(x) = -(x-2)\cdot(x-4)$
$g(x) = (x+3)^2$
$h(x) = (x-3)^2+2$

no zeroes

(2,0)   (4,0)

(-3,0)
one zero

two zeroes

Zeroes occur where the y-coordinate of the parabola
is equal to zero.

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To find the zeroes algebraically, we set y = 0 and solve for the x-values that make the equation true.

Recall: Zero multiplied by anything is zero.

∴ If *(a)(b) = 0* then *a = 0* or *b = 0* (or *both are zero*).

Ex.1 Determine the zero(es) of each by setting y = 0 and solving for x.

(a) y = x(x - 10)

set $y = 0$

$0 = x(x-10)$

$x = 0$ or $x - 10 = 0$
$\qquad\qquad\quad x = 10$

zeroes are $x = 0, x = 10$

(b) y = x(30 - x)

Set $y = 0$

$0 = x(30-x)$

$\boxed{x = 0}$ or $30 - x = 0$
$\qquad\qquad\qquad \boxed{30 = x}$

∴ zeroes are $x = 0$ and $x = 30$

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(c) y = -2(x - 5)(3x - 1)

set $y = 0$

$0 = -2\underbrace{(x-5)}_{0}\underbrace{(3x-1)}_{0}$
$\qquad\qquad\quad$ or

$x - 5 = 0$ or $3x - 1 = 0$
$\boxed{x = 5}\qquad\qquad 3x = 1$
$\qquad\qquad\qquad \boxed{x = \frac{1}{3}}$

to check,
sub x into eqn
and $y = 0$

(d) y = 2(x - 2)^2

set $y = 0$

$0 = 2(x-2)^2$

$0 = 2(x-2)(x-2)$

$x - 2 = 0$ or $x - 2 = 0$
$\quad x = 2 \qquad\qquad x = 2$

one zero
(double root)

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The zeroes can be used to find the <u>vertex</u> (h, k). (x, y)

For the x-coordinate (h), find the midpoint of the zeroes:

$$MP = \frac{x_1 + x_2}{2} = \frac{s + t}{2}$$

For the y-coordinate (k), substitute the midpoint into the equation and solve for y:

$$y = a(x - s)(x - t)$$

$$y = a(MP - s)(MP - t)$$

$$y = \underline{\hspace{3cm}}$$

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Ex.2 Determine the vertex:

(a) y = -(x - 2)(x - 8)

① find zeroes
set y = 0
$0 = -(x-2)(x-8)$
$x-2=0$ or $x-8=0$
$\boxed{x=2}$ $\boxed{x=8}$

② MP of zeroes
$MP = \frac{2+8}{2}$
$= 5$

③ Sub x=5 into eqn
$y = -(5-2)(5-8)$
$= -(3)(-3)$
$\boxed{y=9}$

∴ vertex is $(5, 9)$

(b) y = (3 + x)(2 - x)

set y = 0
$0 = (3+x)(2-x)$
$\boxed{x=-3}$ or $\boxed{x=2}$

$MP = \frac{-3+2}{2}$
$= -0.5$

Sub x=-0.5
$y = (3+(-0.5))(2-(-0.5))$
$= (2.5)(2.5)$
$y = 6.25$

Apr 18-12:03 AM

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Assigned Work:

p. 267 # 7, 9, 10, 11, 15