Brainstorm your answers to the following.

A) Given three points, how would you determine:
1. What type of triangle you have (equilateral, isosceles, scalene)
2. If it is a right triangle.

B) Given 4 points, what is sufficient information to determine if the object is a:
1. Parallelogram
2. Rectangle
3. Rhombus
4. Square

Mar 4-10:12 PM

Geometric Properties of Triangles & Quadrilaterals

March 10, 2010

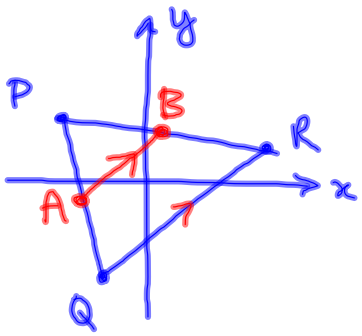
We are going to analyze some geometric theorems.
To do this, we will use the following tools:

- equation of a line (y=mx+b, x=a, y=b)
- slope, parallel/perpendicular lines
- distance formula
- midpoint formula

General Format of Solution:

1. Draw a sketch.
2. State what information is known (given).
3. State what information must be determined.
4. Make a plan of what needs to be done.

Ex.1. A triangle has vertices at P(-2, 2), Q(-1, -3), and R(4, 1). Show that the line segment joining the midpoints of PQ and PR is parallel to QR and half of its length.



Need

~~MP~~ of PQ → A

~~MP~~ of PR → B

slope of AB

slope of QR

distance AB

distance QR

① MP of PQ (A)

$x_m = \frac{x_1+x_2}{2}$ $= \frac{-2+(-1)}{2}$ $= -\frac{3}{2}$

$y_m = \frac{y_1+y_2}{2}$ $= \frac{2+(-3)}{2}$ $= -\frac{1}{2}$

② MP of PR (B)

$x_m = \frac{-2+4}{2}$ $= 1$

$y_m = \frac{2+1}{2}$ $= \frac{3}{2}$

③ slope of AB

$m = \frac{y_2-y_1}{x_2-x_1}$ $= \frac{\frac{3}{2}-(-\frac{1}{2})}{1-(-\frac{3}{2})}$ $= \frac{\frac{3}{2}+\frac{1}{2}}{\frac{2}{2}+\frac{3}{2}}$ $= \frac{2}{\frac{5}{2}}$ $= 2 \times \frac{2}{5}$ $= \frac{4}{5}$

④ slope of QR

$m = \frac{y_2-y_1}{x_2-x_1}$ $= \frac{1-(-3)}{4-(-1)}$ $= \frac{4}{5}$

∴ AB // QR

↑ "parallel to"

Mar 9-6:51 PM

Ex.1. A triangle has vertices at P(-2, 2), Q(-1, -3), and R(4, 1). Show that the line segment joining the midpoints of PQ and PR is parallel to QR and half of its length.

Need

✓ MP of PQ → A

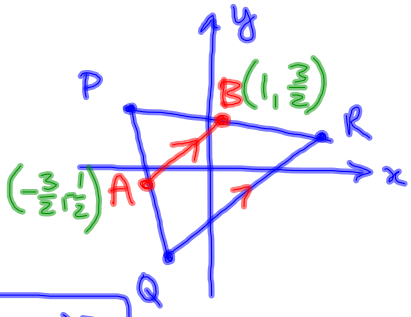
✓ MP of PR → B

✓ slope of AB

✓ slope of QR

distance AB

distance QR



⑤ distance AB

$$d = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2}$$
$$= \sqrt{(1.5-(-0.5))^2 + (1-(-1.5))^2}$$
$$= \sqrt{(2)^2 + (2.5)^2}$$
$$= \sqrt{4 + 6.25}$$
$$= \sqrt{10.25}$$
$$= \sqrt{\frac{41}{4}}$$
$$= \frac{\sqrt{41}}{\sqrt{4}}$$
$$= \frac{\sqrt{41}}{2}$$

$10.25 \times \frac{4}{4} = \frac{41}{4}$

$\div 3.20$

$\sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$

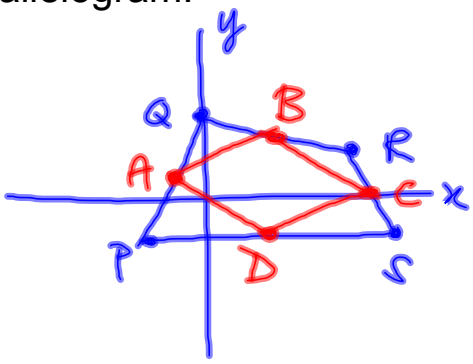
⑥ distance of QR.

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$
$$= \sqrt{(4-(-1))^2 + (1-(-3))^2}$$
$$= \sqrt{(5)^2 + (4)^2}$$
$$= \sqrt{25+16}$$
$$= \sqrt{41}$$
$$\div 6.40$$

∴ AB is half as long as PQ.

Mar 9-6:51 PM

Ex.2. Show that the mid segments of the quadrilateral with vertices at P(-2, -2), Q(0, 4), R(6, 3), and S(8, -1) form a parallelogram.



steps/plan?

① find midpoints A, B, C, D.

② find slope of AB, BC, CD and AD

[③ distance of AB, BC, CD and AD] unnecessary

① MP of PQ (A) A(-1, 1)

$x_A = \frac{-2+(0)}{2} = \frac{-2}{2} = -1$

$y_A = \frac{-2+4}{2} = \frac{2}{2} = 1$

MP of QR (B)

$x_B = \frac{0+6}{2} = 3$

$y_B = \frac{4+3}{2} = \frac{7}{2}$ $B(3, \frac{7}{2})$

MP of RS (C)

$x_C = \frac{6+8}{2} = 7$ $y_C = \frac{3+(-1)}{2} = 1$ $C(7, 1)$

MP of PS (D)

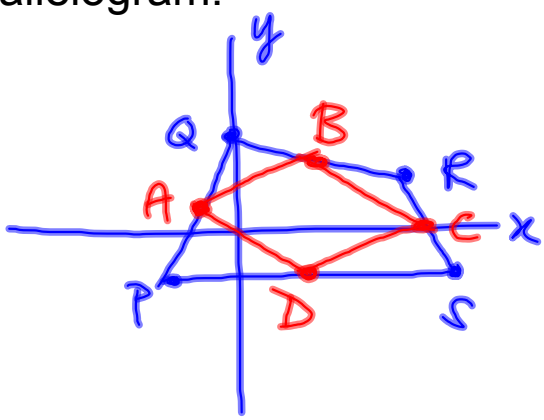
$x_D = \frac{8+(-2)}{2} = 3$ $y_D = \frac{-1+(-2)}{2} = -\frac{3}{2}$ $D(3, -\frac{3}{2})$

Mar 9-6:51 PM

Ex.2. Show that the mid segments of the quadrilateral with vertices at P(-2, -2), Q(0, 4), R(6, 3), and S(8, -1) form a parallelogram.

$A(-1, 1)$ $B(3, \frac{7}{2})$

$C(7, 1)$ $D(3, -\frac{3}{2})$



② slope

$$m_{AB} = \frac{\frac{7}{2} - 1}{3 - (-1)} = \frac{\frac{7}{2} - \frac{2}{2}}{4} = \frac{\frac{5}{2}}{4} = \frac{5}{8}$$

$$m_{BC} = \frac{1 - \frac{7}{2}}{7 - 3} = \frac{\frac{2}{2} - \frac{7}{2}}{4} = \frac{-\frac{5}{2}}{4} = -\frac{5}{8}$$

$$m_{CD} = \frac{-\frac{3}{2} - 1}{3 - 7} = \frac{-\frac{3}{2} - \frac{2}{2}}{-4} = \frac{-\frac{5}{2}}{-4} = \frac{5}{8}$$

$$m_{AD} = \frac{-\frac{3}{2} - 1}{3 - (-1)} = \frac{-\frac{3}{2} - \frac{2}{2}}{4} = \frac{-\frac{5}{2}}{4} = -\frac{5}{8}$$

$\therefore$ AB // CD and BC // AD

$\therefore$ ABCD is a parallelogram.

Mar 9-6:51 PM

Assigned Work:

p.203 # 5, 6, 10

Mar 9-6:51 PM