Finding the Vertex using Symmetry & the y-intercept

Apr. 26, 2010

Recall: 1. factored form: $y = a(x - s)(x - t)$
2. standard form: $y = ax^2 + bx + c$

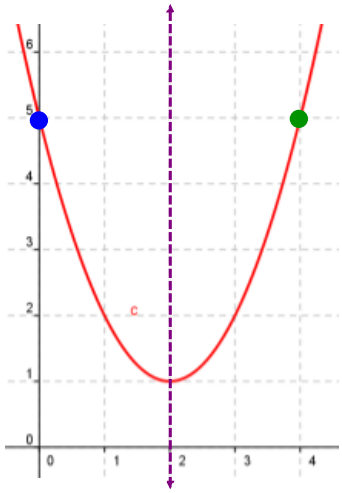
In factored form, s and t are the zeroes.

In standard form, c is the y-intercept.

To find the vertex from standard form, we normally factor and solve for the zeroes.

Apr 19-7:41 PM

This graph has the equation
$y = x^2 - 4x + 5$ (y-int = 5)



In factored form...
... it cannot be factored.
∴ there are no zeroes.

How can we find the vertex algebraically?

Note that the y-intercept (0,5) has a "matching point" on the other side of the axis of symmetry, at (4,5).

We can use the y-intercept and its matching point to find the vertex of the parabola (just like using the zeroes).

Apr 22-8:51 PM

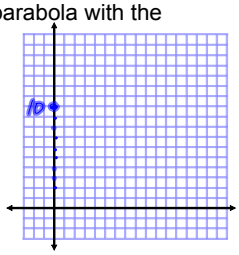
Ex.1 Determine the vertex of the parabola with the equation $y = x^2 - 6x + 10$.

⇒ opens up, y-int at (0,10)

By symmetry, there is a matching point which also has a y-value of 10.

What is the x-value of that point?

⇒ set y = 10 and solve for x



$10 = x^2 - 6x + 10$

$-10 \quad -10$

$0 = x^2 - 6x$

$0 = x(x-6)$

$x = 0$ or $x = 6$

y is 10 when $x = 0$ (y-int) or $x = 6$ (matching point)

∴ matching point (6,10)

x_v = MP of 0 and 6

$x_v = \frac{0+6}{2}$

$= 3$

sub $x = 3$ into eqn

$y = (3)^2 - 6(3) + 10$

$= 9 - 18 + 10$

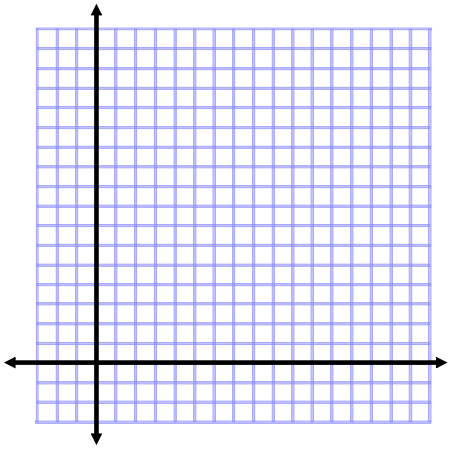
$= 1$

∴ vertex is (3,1)

Apr 22-9:09 PM

The y-int is (0,10) and the matching point is (6,10).

The axis of symmetry must be halfway between the x-values of 0 and 6.



Apr 22-9:09 PM

Ex.2 A ball is thrown into the air. Its height, in metres, after t seconds is $h = -4.9t^2 + 39.2t + 1.75$.

- When does it reach maximum height?
- What is the maximum height?

] vertex?

h-int = 1.75

sub h = 1.75

$1.75 = -4.9t^2 + 39.2t + 1.75$

$0 = -4.9t^2 + 39.2t$

$0 = t(-4.9t + 39.2)$

$t = 0$ or $-4.9t + 39.2 = 0$

$-4.9t = -39.2$

$t = 8$

$t_v = \frac{0+8}{2}$

$= 4$

$h = -4.9(4)^2 + 39.2(4) + 1.75$

$= -4.9(16) + 39.2(4) + 1.75$

$h \doteq 80.2$

∴ max height of 80.2 m after 4s

Apr 22-9:25 PM

Assigned Work:

p. 377 # 5(ii), 8a, 10, 12ab, 15

Test on Wednesday, April 28, 2010

Review Questions:

starting on p.323
1, 5, 6, 7, 9, 10, 11odd, 14odd, 16odd, 19