Unit 1: Linear Systems.

Feb 10/2010

Ways in Which Lines Can Intersect

Each of the following pairs of equations forms a linear system.

Think about their graphs to determine the number of solutions:

number of points where lines cross.

a) $y=2x+4$ $y=2x$

b) $y=2x+4$ $y=-x+4$

c) $y=2x+4$ $y=-x+1$

d) $y=x-3$ $4x-4y=12$

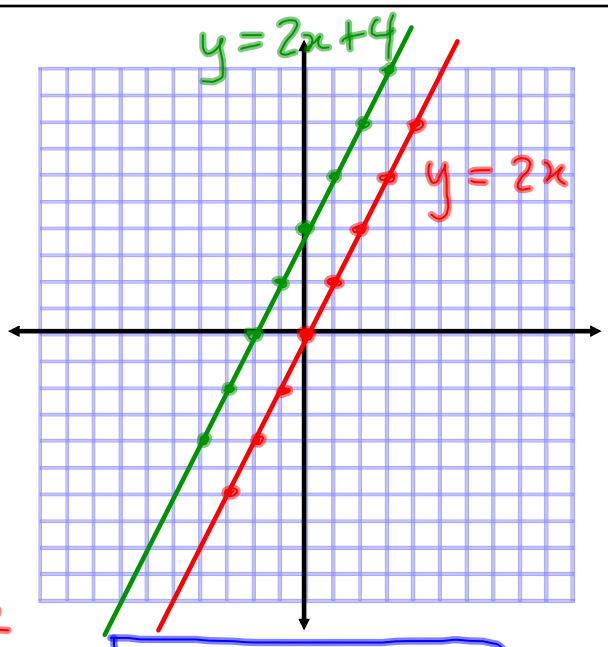
solve graphically

a) $y=2x+4$ ①
$y=2x$ ②

① : y-int at 4
slope, $m=\frac{2}{1}$

② : slope, $m=\frac{2}{1}$
y-int = 0

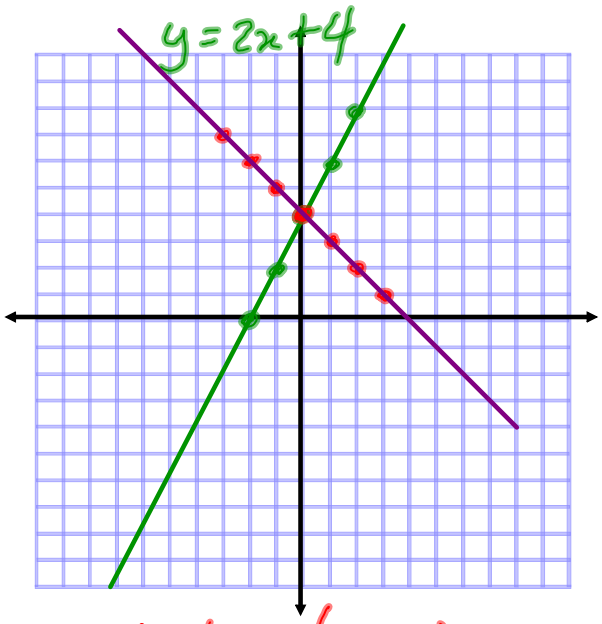
both lines have the same slope
→ parallel lines



no solution
never cross

#1, #2

b) y=2x+4 ①
y=-x+4 ②



① : $m = 2 = \frac{2}{1}$

y-int = 4

② $m = -1 = \frac{-1}{1}$

y-int = 4

lines cross at one point (0,4)

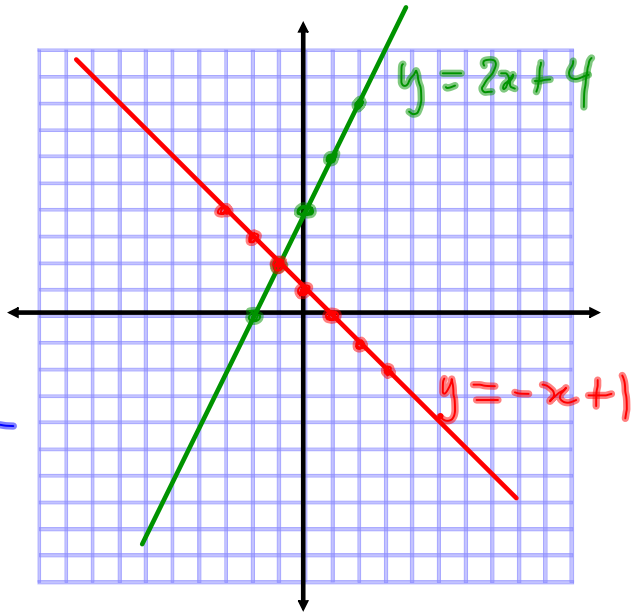
⇒ one solution

Feb 9-8:53 PM

c) y=2x+4 ①
y=-x+1 ②

① : m=2 , b=4

② : m=-1 , b=1



lines cross once at P(-1,2)

→ one solution

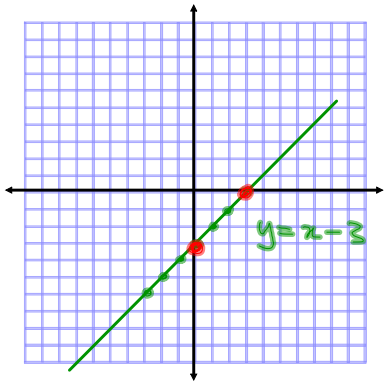
#3, #4

d) y=x-3 ①
4x-4y=12 ②

① : m=1, b=-3

② :
(a) find intercepts
y-int → set x=0
-4y=12
y=-3
x-int → set y=0
4x=12
x=3

(b) rewrite as y=mx+b
4x-4y=12
-4x -4x
$\frac{-4y}{-4} = \frac{-4x}{-4} + \frac{12}{-4}$
y=x-3 → same as equation ①
→ same slope, same y-int



∴ these are the same line!
∴ there are infinite solutions

Feb 9-8:54 PM=

In Summary:

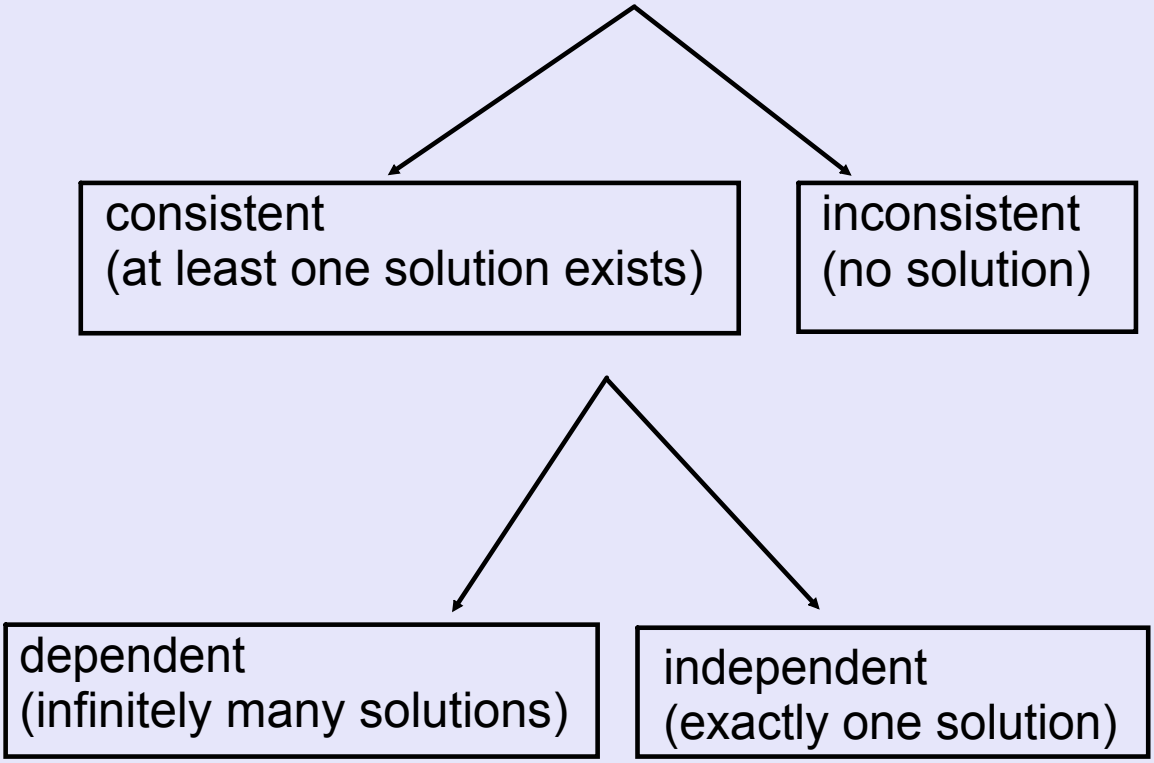
For no solution:
same slope and different y-intercepts.

For exactly one solution:
different slopes only.

For infinitely many solutions:
same slope and same y-intercept.

summary

We classify the systems of equations as follows:



consistent/ inconsistent

Ex1. Determine the number of solutions to this system: (algebraically).

$2x+3y=10$ ①

$10x+15y=50$ ②

① : $2x+3y=10$

$-2x \quad -2x$

$\frac{3y}{3} = \frac{-2x}{3} + \frac{10}{3}$

$y = -\frac{2}{3}x + \frac{10}{3}$

② : $10x+15y=50$

$15y = -10x+50$

$y = \frac{-10}{15}x + \frac{50}{15}$

$y = -\frac{2}{3}x + \frac{10}{3}$

"since" ∴ same slope and same y-int

∴ we have infinite solutions "therefore"

ex1

Ex.2. What value of a gives a system with no solution?

$x(a-1)-y+6=0$
$2x+y-3=0$

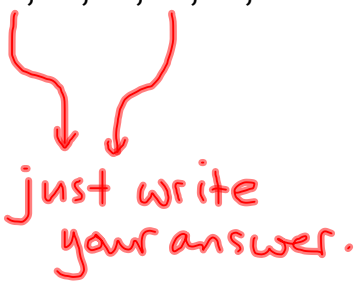
ex2

Ex3. Given $y=2x+5$, write a second equation such that the system has:
i) no solution
ii) exactly one solution
iii) infinitely many solutions.

ex3

Assigned Work:

p. 69 # 1, 3, 5, 7, 8, 13ac



hw

Attachments

Basic 2D Grid.agg