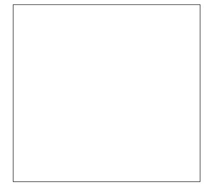
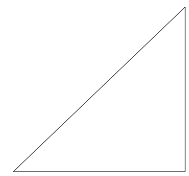
There are two triangles which are frequently used in mathematics to generate exact values for the trigonometric ratios. This is possible because their side lengths hold special relationships with each other. These triangles involve the angles of 30°, 45°, and 60° (and 90°, as they are all right-triangles).

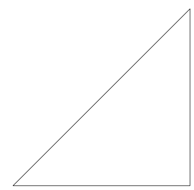
Case 1: The 45°-45°-90° Triangle

1. Start with a square.
2. Set the length of each side to 1.



3. Cutting the square along the diagonal forms a right-triangle.
4. Indicate the right-angle in the figure.
5. Label the sides of length 1.
6. What kind of triangle is this? _ _ _ _ _ _ _ _ _ _ _
7. Considering your answer to the previous question, label the remaining angles.
8. Determine the exact length of the hypotenuse using the Pythagorean theorem.



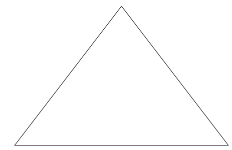


9. Label the hypotenuse (and all other sides & angles)
10. Write the primary trigonometric ratios for 45° using the side lengths of the triangle.

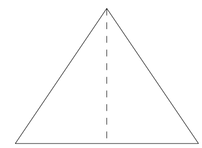
11. Rewrite the primary trignometric ratios with *rationalized denominators*.

Case 2: The 30°-60°-90° Triangle

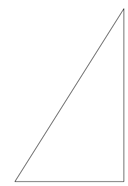
1. Start with an equilateral triangle.
2. What are the angles at each vertex? _ _ _ _ _ _
3. Label each angle.
4. Set the length of each side to 2.



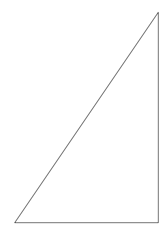
5. Divide the triangle into two identical triangles using a vertical line (the altitude).
6. The base of the triangle is now divided into equal portions of length _ _ _ _ .



7. Consider only one of the new triangles.
8. Indicate the right-angle in the new triangle.
9. Label any sides and angles that are currently known.
10. What is the third angle in the new triangle? _ _ _ _ _ _
11. Label the third angle.



12. Determine the <u>exact</u> height (*altitude*) using the Pythagorean theorem.



13. Label the altitude (and all other sides & angles).
14. Write the primary trigonometric ratios for 30° and 60° using the side lengths of the triangle.

15. Rewrite the primary trignometric ratios with *rationalized denominators*.

Case 3: Multiples of 90°

We have considered angles between 0° and 360°, and considering coterminal angles, even beyond those values. In all cases thus far, we have been able to relate angles back to simple acute right-triangles when determining the primary trigonometric ratios.

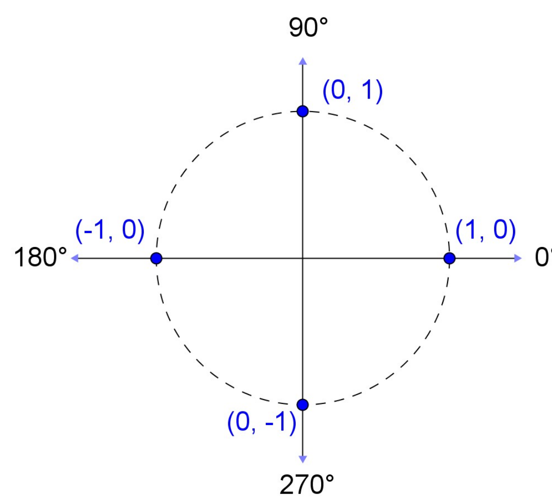
What about the right-angles? Our typical thinking of trigonometric ratios ignores 90°.
What about 180°? How can we determine trignometric ratios of a straight line?

To address these questions, and other angles which are multiples of 90°, consider angles in the cartesian plane and the associated definitions for the primary trigonometric ratios:

$\sin\theta=\frac{y}{r}$ $\cos\theta=\frac{x}{r}$ $\tan\theta=\frac{y}{x}$ where $r^2=\sqrt{x^2+y^2}$

For simplicity, choose points that are always a distance of one (1) unit from the origin. In other words, the points lie on a circle of radius 1 (i.e., $r=1$).

Using the cartesian definitions for the primary trigonometric ratios and the provided figure, determine the primary trigonometric ratios for 0°, 90°, 180°, and 270°.



Practice Questions:

1. The coordinates of a point P on the terminal arm of an angle θ in standard position are given, where $0° \leq \theta \leq 360°$. Draw a diagram to represent this angle, then determine the <u>exact values</u> of $\sin\theta$, $\cos\theta$, and $\tan\theta$.
   a) $P(8,15)$ b) $P(-3,5)$ c) $P(-4,-3)$
   d) $P(12,-5)$ e) $P(-2,-7)$ f) $P(3,-2)$

2. Find the <u>exact value</u> of each trigonometric ratio:
   a) sin 45° b) cos 135° c) tan 225° d) sin 315°
   e) cos 60° f) tan 120° g) sin 300° h) cos 240°
   i) tan 30° j) sin 150° k) cos 210° l) tan 330°

Answers to Practice Questions:

1. a) $\sin\theta = \frac{15}{17}$, $\cos\theta = \frac{8}{17}$, $\tan\theta = \frac{15}{8}$ b) $\sin\theta = \frac{5}{\sqrt{34}}$, $\cos\theta = -\frac{3}{\sqrt{34}}$, $\tan\theta = -\frac{5}{3}$
   c) $\sin\theta = -\frac{3}{5}$, $\cos\theta = -\frac{4}{5}$, $\tan\theta = \frac{3}{4}$ d) $\sin\theta = -\frac{5}{13}$, $\cos\theta = \frac{12}{13}$, $\tan\theta = -\frac{5}{12}$
   e) $\sin\theta = -\frac{7}{\sqrt{53}}$, $\cos\theta = -\frac{2}{\sqrt{53}}$, $\tan\theta = \frac{7}{2}$ f) $\sin\theta = -\frac{2}{\sqrt{13}}$, $\cos\theta = \frac{3}{\sqrt{13}}$, $\tan\theta = -\frac{2}{3}$

2. a) $\frac{1}{\sqrt{2}}$ b) $-\frac{1}{\sqrt{2}}$ c) 1 d) $-\frac{1}{\sqrt{2}}$ e) $\frac{1}{2}$ f) $-\sqrt{3}$
   g) $-\frac{\sqrt{3}}{2}$ h) $-\frac{1}{2}$ i) $\frac{1}{\sqrt{3}}$ j) $\frac{1}{2}$ k) $-\frac{\sqrt{3}}{2}$ l) $-\frac{1}{\sqrt{3}}$