Worksheet **Name:**______________________________________

Piecewise functions

A function is called a *piecewise* function if it has a different algebraic expression for different parts of its domain. A *domain* is a collection of numbers on which the function is defined. Piecewise functions are defined in "pieces" because the function behaves differently on some intervals from the way it behaves on others. The individual pieces of the function may be linear, polynomial, rational or a combination of these. The parts of the domain are usually specified in the form of inequalities.

Since the conditions are different on different parts of the domain, we usually have at least two formulas in a piecewise function. For example, a tax rate schedule is usually stated differently for different ranges of income.

Some examples of piecewise functions

$$f(x) = \begin{cases} 1 & \text{if } x < 2 \\ x+1 & \text{if } x \geq 2 \end{cases}$$

$$f(x) = \begin{cases} x^2 - 1 & \text{if } -1 < x < 3 \\ x+1 & \text{if } x \geq 3 \end{cases}$$

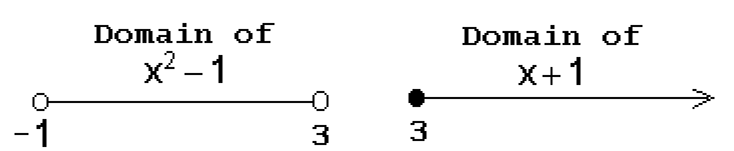
Graphing piecewise functions

To graph a piecewise function, first sketch a picture of each part of the different domains.

For example, to graph

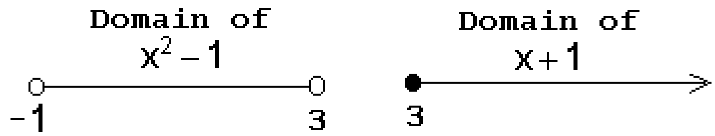
$$f(x) = \begin{cases} x^2 - 1 & \text{if } -1 < x < 3 \\ x+1 & \text{if } x \geq 3 \end{cases}$$

we observe that the domain is specified in the form of inequalities. We can picture these inequalities on a number line as follows,



Open circles at endpoints -1 and 3 indicate that these values are excluded from the domain. The arrow indicates that the domain is not bounded on the right, but continues to include all positive numbers.

After representing the domain inequalities on separate number lines, create a table of values by choosing two or three other values from each separate part of the number line as shown below:

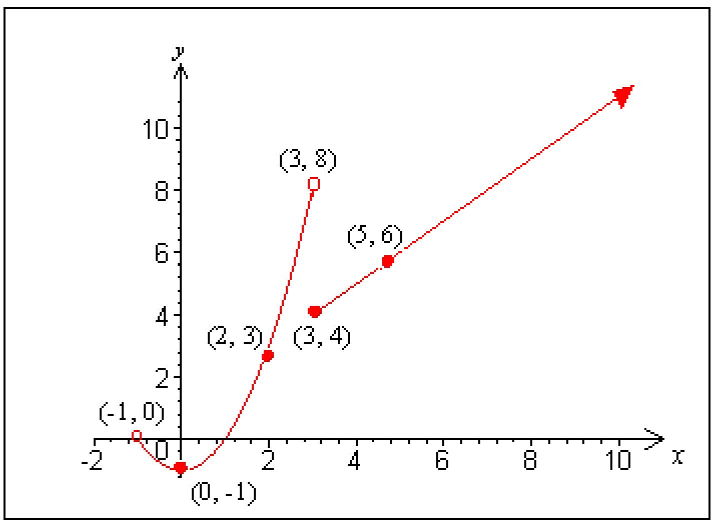


Create a table of values for each function by including the end points and two or three other values from each number line.

	x	$f(x)$
Open endpoints	-1	$(-1)^2 - 1 = 0$
	3	$3^2 - 1 = 8$
Two other values	0	$0^2 - 1 = -1$
	2	$2^2 - 1 = 3$

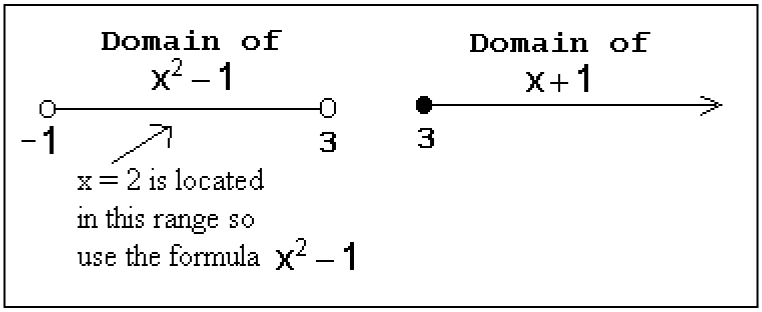
	x	$f(x)$
Closed endpoint	3	$3 + 1 = 4$
Two other values	4	$4 + 1 = 5$
	5	$5 + 1 = 6$

Finally, graph the pair of points from the table.



To find the value of the function at any given value of x, first check the conditions (the domain intervals) from left to right until you find the one that contains the given value of x. Then plug the value of x into the formula associated with that particular domain or condition.

For example to find f(2),



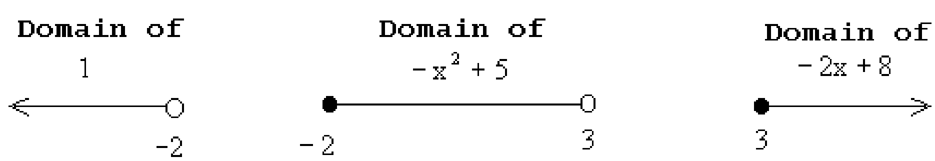
$f(2) = (2)^2 - 1 = 4 - 1 = 3.$

Example

Sketch the graph of

$$f(x) = \begin{cases} -2x + 8, & x \geq 3 \\ -x^2 + 5, & -2 \leq x < 3 \\ 1, & x < -2 \end{cases}$$

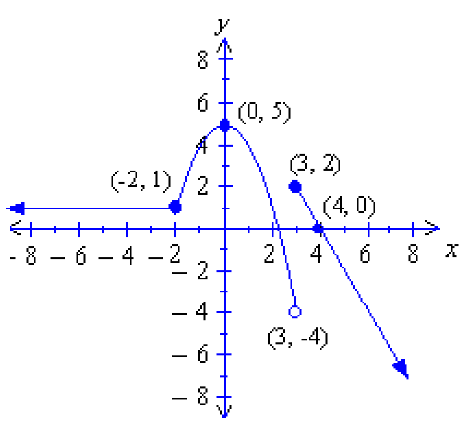
Solution:



	x	$f(x)$
Open endpoint	-2	1
Two other values	-4	1
	-5	1

	x	$f(x)$
Closed endpoint	-2	$-(-2)^2 + 5 = 1$
Open endpoint	3	$-3^2 + 5 = -4$
	0	$-0^2 + 5 = 5$
	2	$-2^2 + 5 = 1$

	x	$f(x)$
Closed endpoint	3	$-2(3) + 8 = 2$
Two other values	4	$-2(4) + 8 = 0$
	5	$-2(5) + 8 = -2$



Problem 1

Sketch the graph of the following functions.

a) f(x) = $\begin{cases} 3x-4, & 2 \le x \le 6 \\ 3x-6, & 6 < x \le 10 \end{cases}$

b) f(x) = $\begin{cases} x, & x \le -1 \\ 1, & -1 < x \le 2 \\ 1-x, & x > 2 \end{cases}$

c) f(x) = $\begin{cases} 2x, & x \le 1 \\ 3-x, & x > 1 \end{cases}$

d) f(x) = $\begin{cases} x^2+1, & x < 3 \\ 2 & 3 \le x < 6 \\ 1-2x, & x \ge 6 \end{cases}$

e) f(x) = $\begin{cases} 1+3x, & -1 \le x \le 3 \\ x^2+1, & 3 < x \le 5 \end{cases}$

Problem 2

a) f(x) = $\begin{cases} 3x-4, & 2 \le x \le 6 \\ 3x-6, & 6 < x \le 10 \end{cases}$ Find f(2) , f(5) and f(6).

b) f(x) = $\begin{cases} x, & x \le -1 \\ 1, & -1 < x \le 2 \\ 1-x, & x > 2 \end{cases}$ Find f(-2) , f(-1), f(2), and f(7).

c) f(x) = $\begin{cases} 2x, & x \le 1 \\ 3-x, & x > 1 \end{cases}$ Find f(-1) , f(5) , f(1), and f(-3).

d) f(x) = $\begin{cases} x^2+1, & x \le 0 \\ 1-2x, & x > 0 \end{cases}$ Find f(2) , f(5) and f(9).

d) f(x) = $\begin{cases} x^2+1, & x < 3 \\ 2 & 3 \le x < 6 \\ 1-2x, & x \ge 6 \end{cases}$ Find f(3) , f(6), f(7) and f(-4).