

Linear Relations and First Differences

NOV. 2/2015

A relation describes how the dependent variable (y) depends on the independent variable (x).

There are four common ways to represent a relation:

- a graph
- a table of values or list of points
- an equation
- a description using words

A linear relation has specific properties that we must learn to test and identify.

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A linear relation has specific properties that we must learn to test and identify.

- (1) Plotting all of the points on a graph will form a straight line.
- (2) The first differences are all equal.
- (3) The equation, when written in the form $y = mx + b$ is only degree 0 (constant) or 1 (in the variable x).

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Ex. Create a table of values for $y = 3x + 2$ and check the first differences.

Note: Check differences of both x and y !

Δx	x	$y = 3x + 2$	$\Delta y = y_2 - y_1$
+1	-2	$3(-2) + 2 = -4$	$(-1) - (-4) = 3$
+1	-1	$3(-1) + 2 = -1$	
+1	0	$3(0) + 2 = 2$	$(2) - (-1) = 3$
+1	1	$3(1) + 2 = 5$	$5 - 2 = 3$
+1	2	$3(2) + 2 = 8$	$8 - 5 = 3$

\therefore all first differences are constant

$\therefore y = 3x + 2$ is a linear relation.

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Ex. Which of the following are linear relations?

(a) $y = -4 - 5x$

(b) $y - x = 9$

(c) $xy + 23 = -3$

(d) $\frac{1}{x} - 9 + y = 0$

(e) $y = x^2 - 7x$

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$$(a) y = -4 - 5x$$

$$y = -5x - 4 \quad \checkmark \text{ linear}$$

deg 1 deg 0

$$(b) y - x = 9$$

$$y = x + 9 \quad \checkmark \text{ linear}$$

1 0

$$(c) xy + 23 = -3$$

-23 -23

$$\frac{xy}{x} = \frac{-26}{x}$$

$$y = \frac{-26}{x} \quad \times \text{ not linear}$$

deg. -1

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Assigned Work:

p. 275 # 1 - 6

desmos

1, 5

6d

2a

4

$$1. (a) y = 5x + 6$$

degree: 1 0

Linear

$$(c) y = 4x^2 + 1$$

degree: 2 0

 \therefore not linear

$$y = mx + b$$

$$x^2 = \frac{1}{x^{-2}}$$

$$(f) y = \frac{6}{x} = 6x^{-1}$$

degree: -1
 \therefore not linear

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2 (a)

Δx	x	y	$\Delta y = y_2 - y_1$
	0	5	
+1	1	6	6-5 = 1
+1	2	8	8-6 = 2
+1	3	12	12-8 = 4

\therefore not linear

Δy not the same

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4.

# houses	# segments
0	1
1	6
2	11
3	16
4	21

\therefore linear

$y = mx + b$

$m = \frac{\Delta y}{\Delta x}$

$= \frac{5}{1}$

$= 5$

what is y when x is 0?

$y = 5x + 1$

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5. (b)

# sides	# diagonals
0	
1	
2	
3	
4	2
5	5
6	9
7	

Δy 's not the same.
 \therefore not linear

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6(d)

x diagram	# toothpicks
0	1 $\rightarrow b=1$
1	4
2	7
3	10
...	
10	

$m = \frac{\Delta y}{\Delta x} = \frac{3}{1} = 3$ $b = 1$
 $y = 3x + 1$

(d) extrapolate to 10th diagram.

set $x = 10$

$$y = 3(10) + 1$$

$$= 30 + 1$$

$$= 31$$

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