Quadratic Relations in Factored Form

Key Concepts:

- factored form of quadratic relation
- direction of opening from 'a'
- solving for zeroes

- using symmetry to find:
  - x-coordinate of vertex
  - axis of symmetry

- using substitution to find:
  - y-coordinate of vertex
  - y-intercept

Apr 10-6:32 PM

Is $y = 2(x+1)(x-5)$ a quadratic relation?

Examine 1st and 2nd differences:

x	y	Δy	$\Delta^2 y$
-2	14		
		-14	
-1	0		4
		-10	
0	-10		4
		-6	
1	-16		4
		-2	
2	-18		

constant

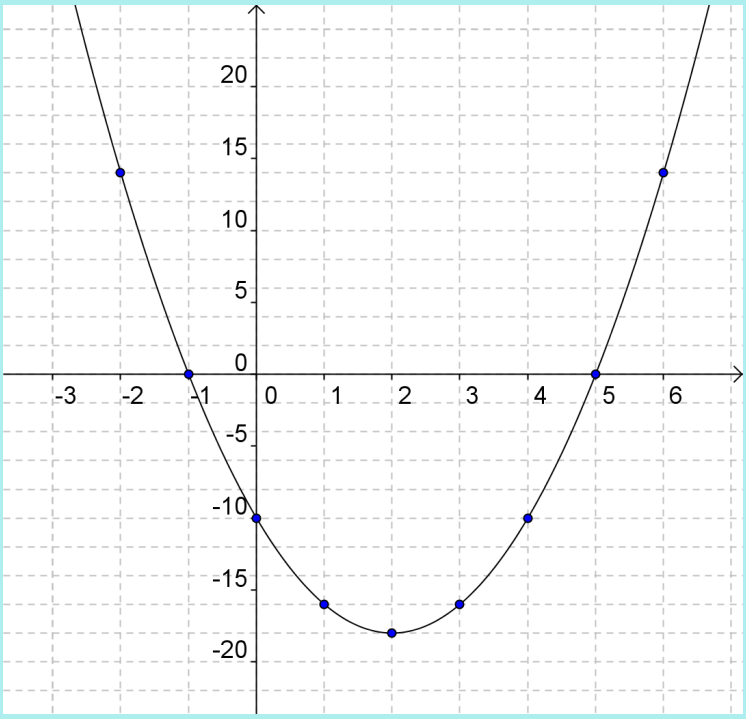
$\therefore$ quadratic

Oct 19-8:29 PM

Is $y = 2(x+1)(x-5)$ a quadratic relation?

Graph the relation:

x	y
-2	14
-1	0
0	-10
1	-16
2	-18
3	-16
4	-10
5	0
6	14



Oct 19-8:29 PM

Quadratic Relations in Factored Form

March 23/2016

The equation of a quadratic relation may be written in several forms:

1. standard form: $y = ax^2 + bx + c$
2. factored form: $y = a(x - s)(x - t)$
3. vertex form: $y = a(x - h)^2 + k$

The factored form, $y = a(x - s)(x - t)$, is most useful for finding the zeroes, which are $x = s$ and $x = t$.

Mar 20 - 4:17 PM

Consider the following...

Give two numbers that have a product of zero:

$0(10)=0$ $(0)(0)=0$

$10(6)=0$ $(5+5)(0)=0$

What do you notice?

only way to get a product of zero is $\times 0$

Mar 31-8:45 AM

Consider the following...

Give two numbers that have a product of zero:

What do you notice? $(\text{any value}) \times 0 = 0$

Solve:

(a) $3x = 0$

$\frac{3x}{3} = \frac{0}{3}$

$x = 0$

(b) $57y = 0$

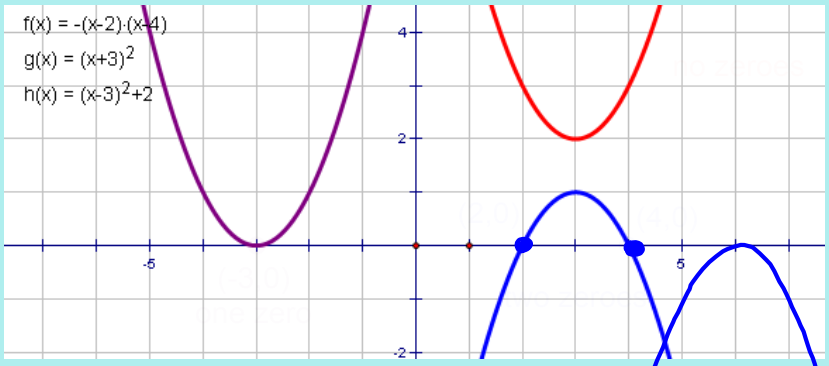
$y = 0$

(c) $3xy = 0$

$x = 0$ OR $y = 0$

Mar 31-8:45 AM

Depending upon the location of the vertex, and whether the parabola opens up or down, it may have 0, 1, or 2 distinct (unique) zeroes.



Zeroes occur where the y-coordinate of the parabola is equal to zero.

Apr 17-11:18 PM

To find the zeroes algebraically, we **set y = 0** and solve for the x-values that make the equation true.

Ex. Determine the zero(es) of each

(a) y = x(x - 10)

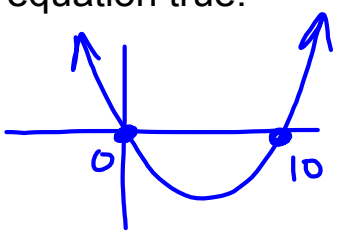
set $y = 0$

$0 = x(x-10)$

$x = 0$ or $x - 10 = 0$

$x = 10$

Recall:



Zero multiplied by anything is zero.

If the product (a)(b) = 0 then:
a = 0,
or b = 0,
or both are zero.

(b) y = -2(x - 5)(3x - 1)

(c) $y = 2(x - 2)^2$

Apr 17-11:30 PM

Ex. Determine the zero(es) of each

(b) $y = -2(x - 5)(3x - 1)$

for zeroes, set $y = 0$

$0 = -2(x-5)(3x-1)$

$x - 5 = 0$ or $3x - 1 = 0$

$x = 5$ or $3x = 1$

$x = \frac{1}{3}$

(c) $y = 2(x - 2)^2$

set $y = 0$

$0 = 2(x-2)^2$

$0 = 2(x-2)(x-2)$

$x - 2 = 0$ or $x - 2 = 0$

$x = 2$ or $x = 2$

Apr 17-11:30 PM

The zeroes and symmetry can be used to find the vertex (h, k).

For the x-coordinate (h), find the midpoint of the zeroes:

$$MP_x = \frac{x_1 + x_2}{2} = \frac{s + t}{2}$$

For the y-coordinate (k), substitute the midpoint into the equation and solve for y:

$y = a(x - s)(x - t)$

$y = a(MP_x - s)(MP_x - t)$

Ex. Determine the vertex using the zeroes.

$y = -2(x - 2)(x - 8)$ predict zeroes:

Apr 17-11:45 PM

Ex. Determine the vertex using the zeroes.

$y = -2(x - 2)(x - 8)$ predict zeroes: $2, 8$

Set $y = 0$

$0 = -2(x-2)(x-8)$

$x-2=0$ $\quad x-8=0$

$x=2$ $\quad x=8$

$MP_x = \frac{2+8}{2}$

$= 5$ → x-coordinate of vertex.

→ axis of symmetry $x = 5$

Sub $x=5$: $y=-2(5-2)(5-8)$

$y=-2(3)(-3)$

$y = 18$ $\quad \therefore V(5, 18)$

Apr 18-12:03 AM

Ex. A parabola has zeroes at -3 and 2, and a y-intercept of 18. Determine the equation.

$(0, 18)$

$x \quad y$

$y = a(x-s)(x-t)$

$y = a(x+3)(x-2)$

sub $(0, 18)$

$18 = a(0+3)(0-2)$

$18 = a(3)(-2)$

$\frac{18}{-6} = \frac{-6a}{-6}$

$a = -3$

$y = -3(x+3)(x-2)$

Oct 19-9:26 PM

Assigned Work:

p. 155-157 # 2, 3, 4ace, 5, 6ace, 7, 10

3. $y = a(x-r)(x-s)$

$x-(-6) = x+6$

zeroes: $(2,0)$ and $(-6,0)$ (r, s)

$P(3,5)$

$y = a(x-2)(x+6)$

Sub $P(3,5)$ (x, y)

$5 = a(3-2)(3+6)$

$5 = a(1)(9)$

$\frac{5}{9} = \frac{9a}{9}$

$a = \frac{5}{9}$

Set $x=0$, find y-int

Set $y=0$, find x-int(s)

4(c) $y = (x-2)(x-2)$

y-int, set $x=0$: $y = (0-2)(0-2)$

$= (-2)(-2)$

$= 4$

zeroes (x-int)

set $y=0$: $0 = (x-2)(x-2)$

x-coordinate of zero

$x-2=0$ or $x-2=0$

$x=2$ $x=2$

Equation of AofS: $x=2$

↑ equation of a vertical line.

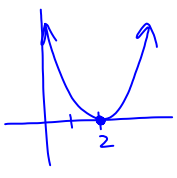
for vertex: Sub x from AofS

$y = (2-2)(2-2)$

$= (0)(0)$

$= 0$

$\therefore V(2,0)$



Mar 24-2:01 PM

6(e) $y = a(x-r)(x-s)$

$V(5,0)$ y-int $(0,-10)$

only zero
or
both zeroes are 5

$y = a(x-5)(x-5)$

$y = a(x-5)^2$

Sub $(0,-10)$: $-10 = a(0-5)^2$

$-10 = a(-5)^2$

$\frac{-10}{25} = \frac{25a}{25}$

$a = \frac{-10^{2}}{25_{5}}$

$a = -\frac{2}{5}$

$y = -\frac{2}{5}(x-5)^2$

Mar 24-2:10 PM