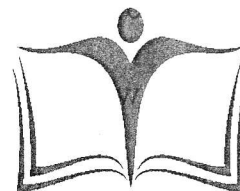


MPM2D
Examination
Exam B, 2013
Length: 3 hours
(Exam set for 2 hrs. + 1 hr. flex time)



OTTAWA-CARLETON
DISTRICT SCHOOL BOARD

Name : _____
Teacher : _____
School : _____

Instructions to students:

1. This examination booklet is **12 pages** long.
Please check that you have all the pages.
2. Answer all questions with complete solutions in the spaces provided on the examination paper.
3. You may use any school-approved calculator on this examination.
Make sure that your calculator is in **DEGREE** mode.
Do not share your calculator.
4. There is a formula sheet that goes with the examination.
5. Diagrams are not drawn to scale.

A) Trigonometry

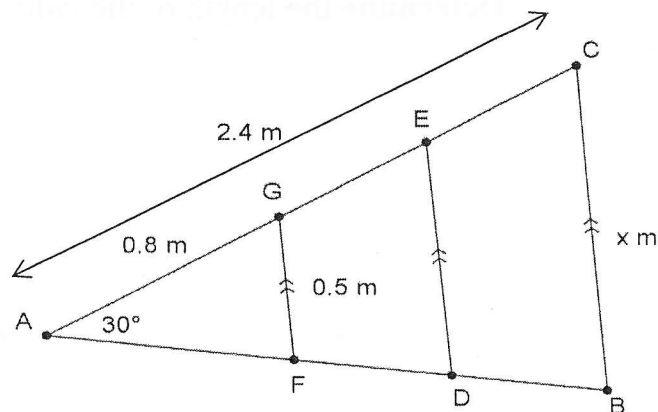
- A1) Roof trusses are being constructed as shown in the diagram. Determine the value of x .

$\triangle GAF \sim \triangle CAB$ by AA~

$$\therefore \frac{CB}{GF} = \frac{CA}{GA}$$

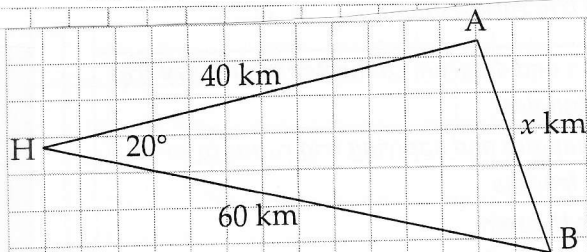
$$\frac{x}{0.5} = \frac{2.4}{0.8}$$

$$x = 1.5$$



\therefore the value of x is 1.5 m.

- A2) Two boats leave a harbour at the same time in directions that are 20° apart. If one is travelling at 20 km/h and the other at 30 km/h, how far apart are they after two hours?



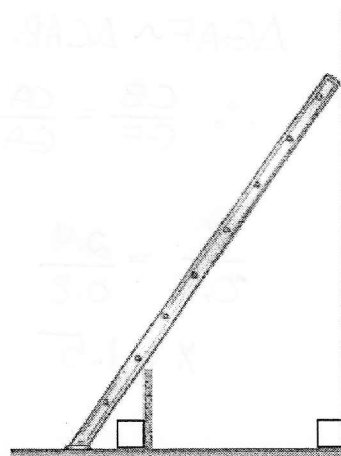
$$x^2 = 40^2 + 60^2 - 2(40)(60)\cos 20^\circ$$

$$x^2 = 689.48$$

$$x = 26.3$$

The boats are approximately 26.3 km apart after two hours.

- A3) A ladder must reach 6 metres up a wall and pass over a fence. The fence is 1.5 metres in height and 3 metres from the wall. Determine the length of the ladder.



Method 1: Solve for the angle:

$$\tan \theta = \frac{4.5}{3}$$

$$\theta \doteq 56.3^\circ$$

Solve for l_1 :

$$\sin 56.3^\circ = \frac{4.5}{l_1}$$

$$l_1 \doteq 5.4$$

Solve for l_2 :

$$\sin 56.3^\circ = \frac{1.5}{l_2}$$

$$l_2 \doteq 1.8$$

The ladder must be 7.2 m long.

Method 2: Let x represent the base of the small triangle.

Then:

$$\frac{4.5}{3} = \frac{1.5}{x}$$

$x = 1$
(or similar proportional reasoning)

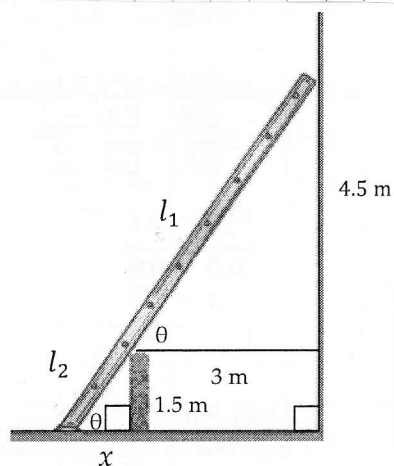
Then, using Pythagoras:

$$l^2 = (6)^2 + (4)^2$$

$$l^2 = 52$$

$$l = \pm\sqrt{52}$$

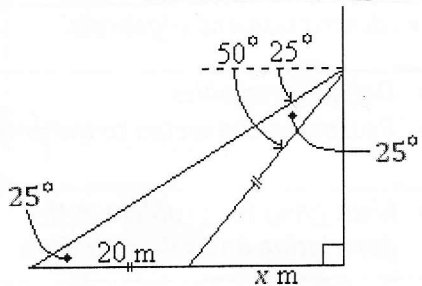
The ladder is about 7.2 m long.



- A4) From a window in an office building, two cars which are 20 metres apart are seen in the same direction, one at an angle of depression of 25° and the other at an angle of depression of 50° .

Your friend thinks that since one angle is half of the other, one car must be twice as far away from the base of the building as the other.

Do you think this is true? **Justify** your answer.

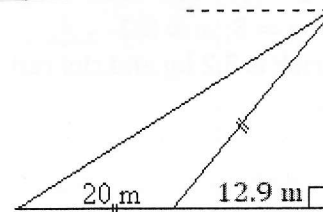


The obtuse triangle is isosceles
 \therefore the hypotenuse of the right triangle is 20 m.

$$\frac{x}{20} = \sin 40^\circ$$

$$x = 20 \sin 40^\circ$$

$$x \approx 12.9$$



In this case, the cars are not twice as far away.

Since the relationship between the angle and the distance is not linear, doubling the angle will not cause the distance to double.

B) Analytic Geometry

let x represent white bricks
let y represent red bricks

- B1) A sample of two white bricks and five red bricks has a total mass of 31.4 kg.
A sample of four white bricks and one red brick has a total mass of 17.8 kg.
Determine the mass of each colour of brick.

$$\textcircled{1} (2x + 5y = 31.4) \times 2$$

$$\textcircled{2} 4x + 1y = 17.8$$

$$\begin{array}{r} \textcircled{1} 4x + 10y = 62.8 \\ - \textcircled{2} 4x + 1y = 17.8 \\ \hline \end{array}$$

$$\frac{9y}{9} = \frac{45}{9}$$

$$y = 5$$

∴ The white brick is 3.2 kg &
the red brick is 5 kg.

$$\text{Sub } y=5 \text{ into } \textcircled{1}$$

$$2x + 5(5) = 31.4$$

$$2x = 31.4 - 25$$

$$\frac{2x}{2} = \frac{6.4}{2}$$

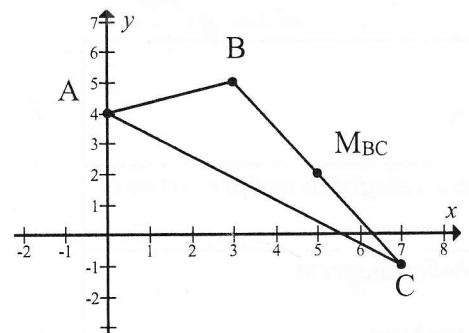
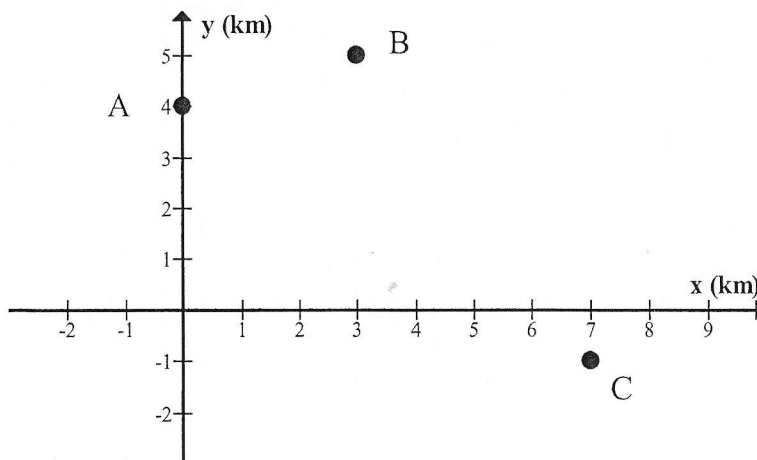
$$x = 3.2$$



- B2) Ahmed, Bonnie and Clyde are located at $A(0, 4)$, $B(3, 5)$ and $C(7, -1)$ respectively. They agree to meet at their truck located halfway between Bonnie and Clyde.

Determine the distance Ahmed must travel to reach the truck.

Note: each unit on the grid represents 1 km.



$$\text{Midpoint BC: } M_{BC}(5, 2)$$

$$|AM_{BC}| = \sqrt{29} \approx 5.4$$

Ahmed must travel
approximately 5.4 km.

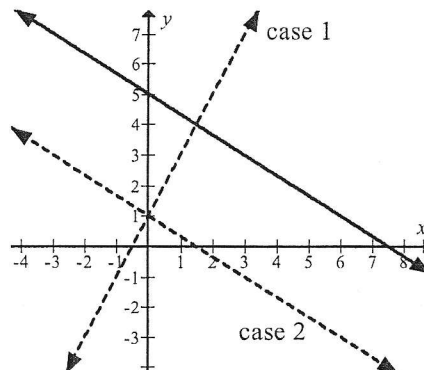
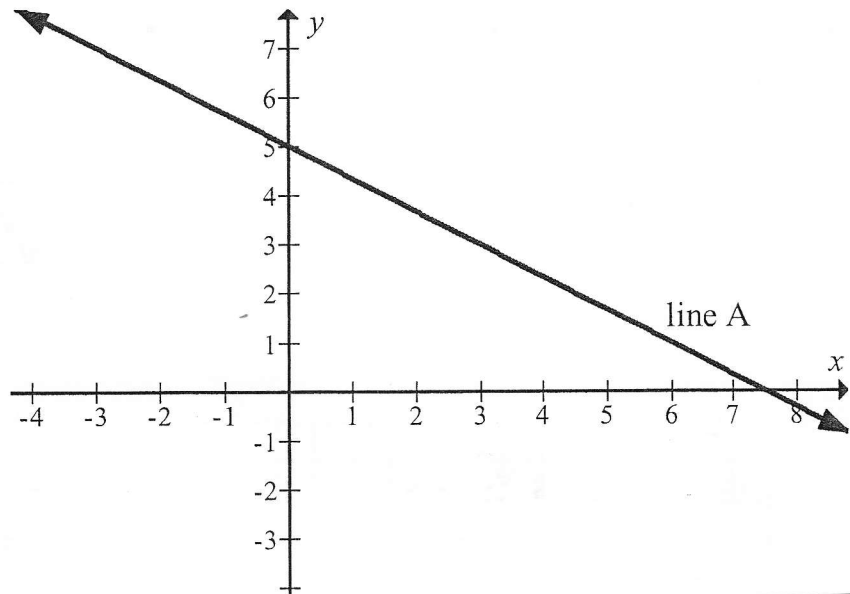
B3) The graph below is one relation in a linear system.

- a) **Determine** an equation of another linear relation that creates a system with no solution.

Justify your choice.

- b) **Determine** other linear relations that create systems with different numbers of solutions.

Justify your choices using multiple representations.



For no solutions, we need equivalent slopes with different y-intercepts or we need an a and b from $ax + by = c$ form which are not multiples of 2 and 3 and c which is not the same multiple of 15

Let $m = -\frac{2}{3}$ and choose any value other than 5 for the y-intercept giving the equation $y = -\frac{2}{3}x + 1$ or $2x + 3y = 3$

b) **Case 1:** One solution

We need a non-equivalent slope or an a and b from $ax + by = c$ form which are not multiples of 2 and 3

Let $m = 2$ and choose any value for the y-intercept giving the equation $y = 2x + 1$.

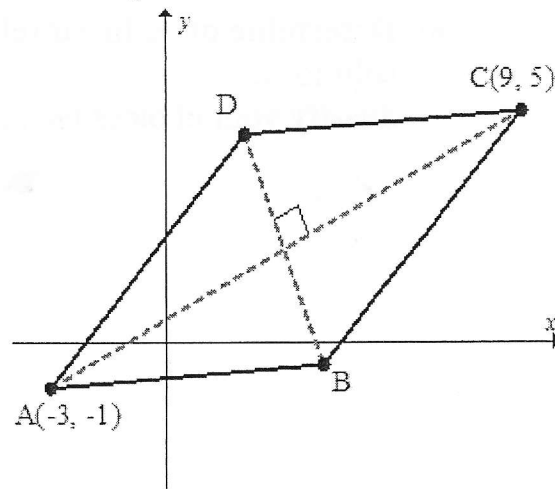
Since $\frac{2}{1} \neq -\frac{2}{3}$, the lines are not parallel and therefore they must intersect.

Case 2: Infinite solutions

We need a line coincident with the given line so we need a , b and c from $ax + by = c$ to be multiples of 2, 3 and 15. Choose $a = 4$, $b = 6$ and $c = 30$ so $4x + 6y = 30$ would represent a line coincident with the given line.

B4) The diagonals of any rhombus are perpendicular bisectors of each other. Line segments AC and BD are diagonals of rhombus ABCD.

- Determine** possible coordinates for B and D.
- Are there other possible locations for B and D? **Explain.**



Part a)

Since AC and BD are perpendicular bisectors, $M_{AC}(3,2)$ is the midpoint of both AC and BD. Therefore, possible values for B and D must be equidistant from $M_{AC}(3,2)$. One example is to let B be 2 units left of $M_{AC}(3,2)$ and D be 2 units right. Then $D(1,6)$ and $B(5,-2)$ could be the endpoints of diagonal BD, ensuring that Quad ABCD is a rhombus.

Part b)

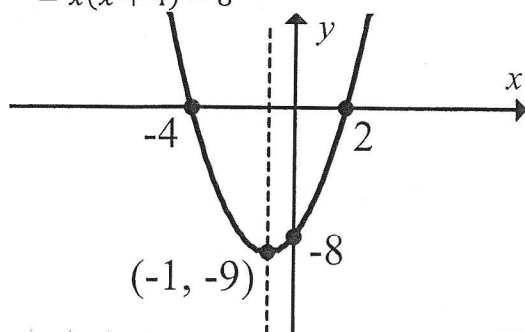
There are an infinite number of possible locations for B and D, as long as they are along the line perpendicular to AC and passing through $M_{AC}(3,2)$. To find equation of BD, we need $m_{AC} = \frac{1}{2}$. Thus, $m_{BD} = -2$. Using this slope and the midpoint gives the equation $y = -2x + 8$.

A square would occur if B and D are the same distance from M_{AC} as A and C. By counting grid squares, the location of B and D that would create a square are $B(0,8)$ and $D(6,-4)$.

C) Quadratic Relations

C1) Determine the key features of the parabola with equation $y = x^2 + 2x - 8$.

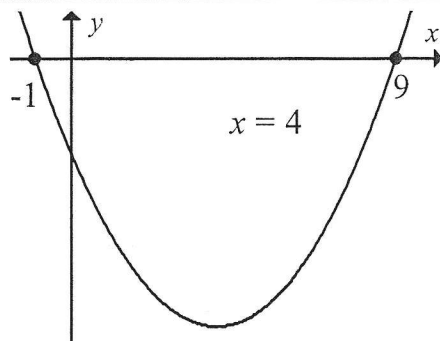
$$\begin{aligned}y &= x^2 + 2x - 8 \\&= (x - 2)(x + 4) \\&= (x + 1)^2 - 9 \\&= x(x + 4) - 8\end{aligned}$$



Key features:

- x-intercepts: 2, -4
- vertex (-1, -9)
- y-intercept -8
- axis of symmetry $x = -1$
- opens upward
- congruent to $y = x^2$ (no vertical stretch or compression)

C2) Determine the equation of a quadratic relation with an axis of symmetry of $x = 4$ and an x-intercept at -1.



I can use symmetry to find the other x-intercept. The difference between the given x-intercept, $x = -1$ and the a.o.s, $x = 4$ is 5. If I add 5 to the a.o.s, I get 9. Therefore an equation will be $y = (x + 1)(x - 9)$. For my equation I have an *a-value* of 1, but this can be any number other than 0.

- C3) The table below shows some data for the average number of hours of sleep per week, by age.

Age	Average number of hours of sleep per week
15	72
16	58
17	48
18	42
19	40
20	42

Vertex

1st

2nd

$$\begin{array}{l}
 58 - 72 = -14 \\
 48 - 58 = -10 \\
 42 - 48 = -6 \\
 40 - 42 = -2 \\
 42 - 40 = 2
 \end{array}
 \quad
 \begin{array}{l}
 -10 - (-14) = 4 \\
 -6 - (-10) = 4 \\
 -2 - (-6) = 4 \\
 2 - (-2) = 4
 \end{array}$$

- a) **Determine** an equation which could be used to predict the average number of hours of sleep per week for other ages, based on the data in the table.
- b) Could this model be used to predict the average number of hours of sleep per week for all ages? **Explain.**

a) solve a

$$y = a(x - 19)^2 + 40$$

sub in a point (15, 72)

$$72 = a(15 - 19)^2 + 40$$

$$32 = a(-4)^2$$

$$\frac{32}{16} = \frac{16a}{16}$$

$$a = 2$$

$$y = 2(x - 19)^2 + 40$$

b) hours in a week = 7×24
= 168 hours
in a week.

sub 168 into y

$$168 = 2(x - 19)^2 + 40$$

$$168 - 40 = 2(x - 19)^2$$

$$\frac{128}{2} = \frac{2(x - 19)^2}{2}$$

$$\sqrt{64} = \sqrt{(x - 19)^2}$$

$$\pm 8 = x - 19$$

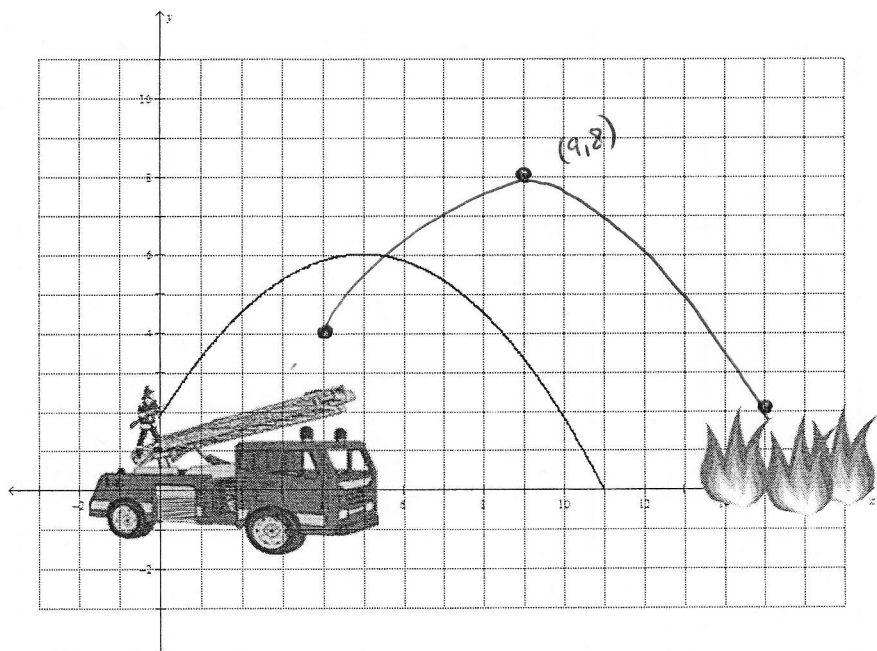
$$x = \pm 8 + 19$$

$$x = 27 \text{ or } 11$$

- C4) A stream of water flowing out of a hose can be modelled by the equation $y = -\frac{1}{6}(x+1)(x-11)$, where y is the height of the water, in metres above the ground, and x is the horizontal distance from the hose, in metres.

The fireman climbs up the inclined ladder so that the peak of the stream is now 4 metres further horizontally and 2 metres higher.

How far can the water stream now reach?



Find the zeroes

$$-\frac{1}{6}(x-9)^2 + 8 = 0$$

$$(x-9)^2 = 48$$

$$x-9 = \pm\sqrt{48}$$

$$x = -\sqrt{48} + 9 \text{ or } x = \sqrt{48} + 9$$

$$x \doteq 2.07 \text{ or } x \doteq 15.93$$

Since 15.93 m is the farthest distance, the water flow will now reach 15.93 m from the hose.