Determining Transformed Functions from Graphs

Sept 12/2016

Recall: Given $y = a\,f\left[k\left(x-p\right)\right]+q$

1. vertical scaling by a for $a \neq 1$
(includes vertical reflection for a <0)

2. horizontal scaling by $\frac{1}{k}$ for $k \neq 1$
(includes reflection)

3. horizontal translation by p

4. vertical translation by q

$f(3x)$ h. compression by 3 ✓
h. stretch by $\frac{1}{3}$ X
© h. compression by $\frac{1}{3}$ text books
h. scaling by $\frac{1}{3}$ ok

Determining Transformed Functions from Graphs

$$y = a\,f\left[k\left(x - p\right)\right] + q$$

$\quad\quad 1 \quad\quad 2 \quad\quad 3 \quad\quad 4$

For any single point, the transformations can be summarized as:

$$\left(x, y\right) \to \left(\frac{x}{k} + p,\ ay + q\right)$$

$\quad\quad 2 \quad\quad 3 \quad\quad 1 \quad\quad 4$

Given two sets of points (before and after transformation), use logic, deductive reasoning, and linear systems of equations to determine values for a, k, p, and q.

$$y = (2x)^2$$

$$= 4x^2$$

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Tips for parabolas: $y = a(x-p)^2 + q$

1. The vertex of the parent function is at (0, 0). The value zero is not affected by scaling (a or k), only translations (p or q). The vertex will be at (p, q).

2. Parabolas can ignore the horizontal scaling, k, because there is an equivalent 'a' value. Set k = 1.

k

$$\boxed{\begin{array}{l} y = (3x)^2 \\ y = 9x^2 \end{array}}$$

a

3. Use the step pattern (1, 3, 5, ...) from the vertex to determine the vertical scaling, 'a'.

Tips for radicals: $y = a\sqrt{k(x-p)} + q$

1. The parent function starts at (0, 0), just like a parabola. The value zero is not affected by scaling (a or k), only translations (p or q).

2. The sign of 'a' and 'k' are both important for reflections.

3. Use 'k' for scaling. The horizontal scaling is more likely to give a "nice" (integer) value.

Set a to -1 or +1 (depending on v. reflection). Solve for k.

$$(x, y) \rightarrow \left(\frac{x}{k} + p,\ ay + q\right)$$

$$y = 3\sqrt{x} \qquad\qquad y = \sqrt{3x}$$

$$= \sqrt{9x} \qquad\qquad = \sqrt{3}\sqrt{x}$$

Tips for rationals: $y=\dfrac{a}{k\left(x-p\right)}+q$

1. The parent function has asymptotes at x=0 and y=0.

   The new asymptotes will be at x = p and y = q.

2. Use only <u>one of</u> 'a' or 'k' for scaling and reflection.

$$y=\frac{1}{3\left(x-2\right)}\Leftrightarrow y=\frac{1}{3}\times\frac{1}{x-2}$$

$k=3$ $\qquad$ $a=\dfrac{1}{3}$

'a' is usually easier to work with, so set k = 1.

Assigned Work:

f ?

p.36 # 4, 5, 6, 7, 9, 10, 15, 16ab

b b

d

4. $(x, y) \rightarrow \left(\frac{x}{k} + p, ay + q\right)$

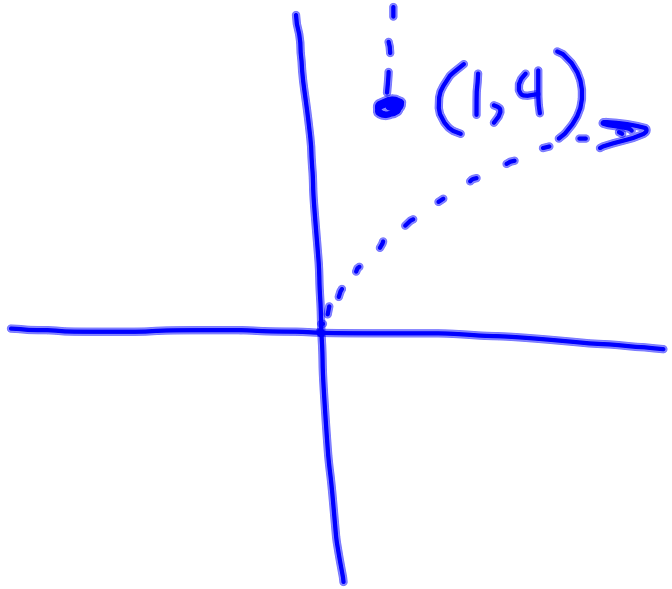
(b) $y = f(x-3)$ 3 units right

$p = 3$

$(2,3) \rightarrow (5,3)$

10. $f(x) = \sqrt{x}$

(b) $h(x) = 2f(x-1) + 4$



$D = \{x \in \mathbb{R} \mid x \geq 1\}$

$R = \{y \in \mathbb{R} \mid y \geq 4\}$

9(f) $P(1,8)$ on $y = f(x)$

$$y = \underset{a}{0.5}\, f\Big[\underset{k}{0.5}\big(\overset{x-p}{x+3}\big)\Big] \underset{q}{+3} \qquad p=-3$$

$$(1,8) \xrightarrow{a} (1,4) \xrightarrow{k} (2,4) \longrightarrow (-1,4) \longrightarrow (-1,7)$$

$$(x,y) \longrightarrow \left(\frac{x}{k}+p,\ ay+q\right)$$

15. $(3,6)$ on $y=2f(x+1)-4$

$$(x,y) \rightarrow \left(\frac{x}{k}+p,\ ay+q\right)$$

$$\frac{x}{k}+p = 3 \qquad ay+q = 6$$

$$\frac{x}{1}-1=3 \qquad 2y-4=6$$

$$\frac{x}{1}=4 \qquad 2y=10$$

$$x=4 \qquad y=5$$