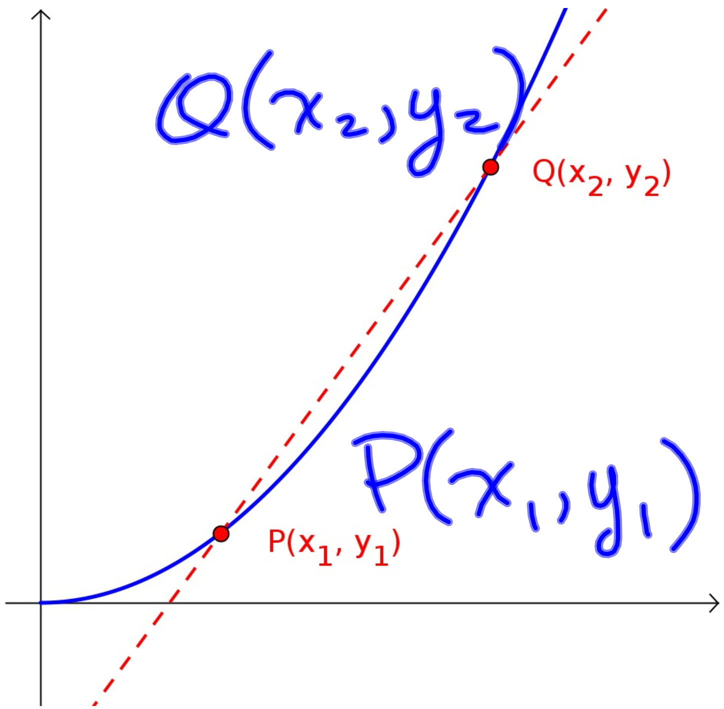
Sep.16/2016

Rates of Change

Given the graph of a function, the <u>average rate of change</u> is defined as the slope of the <u>secant line</u> between two points.



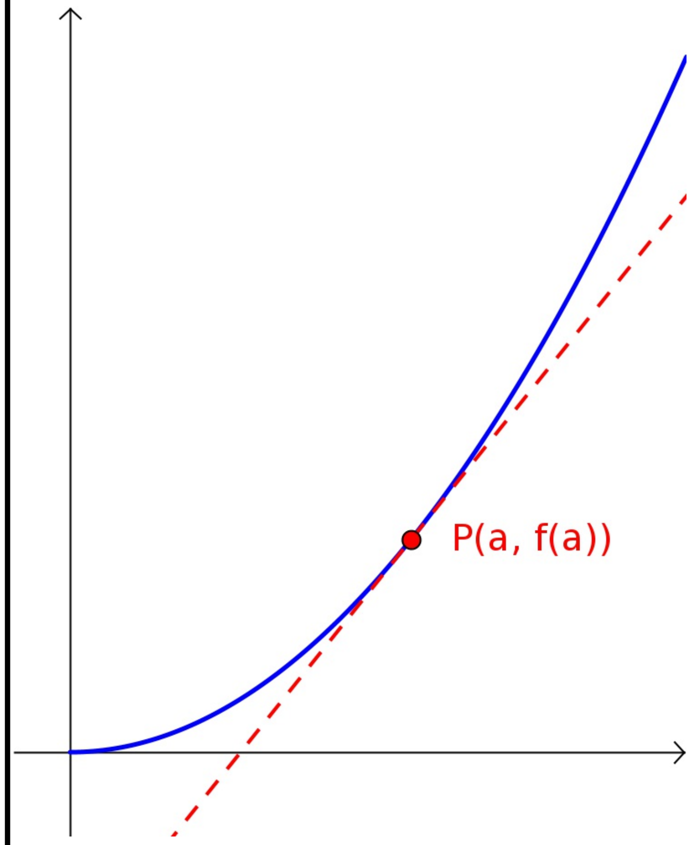
$$\text{avg RoC} = m_{PQ} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Using function notation, the points can also be written:

$$P(x_1, f(x_1))$$
$$Q(x_2, f(x_2))$$

$$\therefore \text{avg RoC} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

The instantaneous rate of change is the slope of the tangent line at a particular point of interest, defined by a specified value of the independent variable (e.g., at x = a).



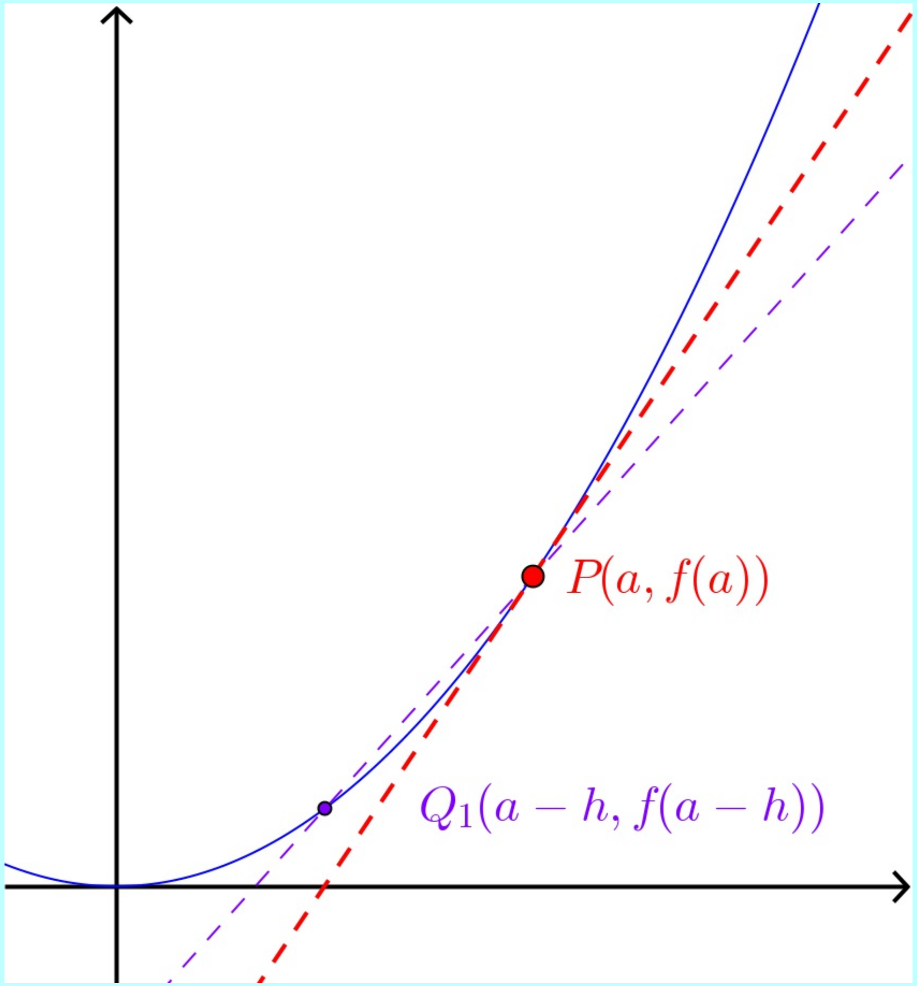
For now, we can only estimate this value by determining the average rate of change over a very small interval near x = a.

(a) a preceding interval uses a point before the point of interest.

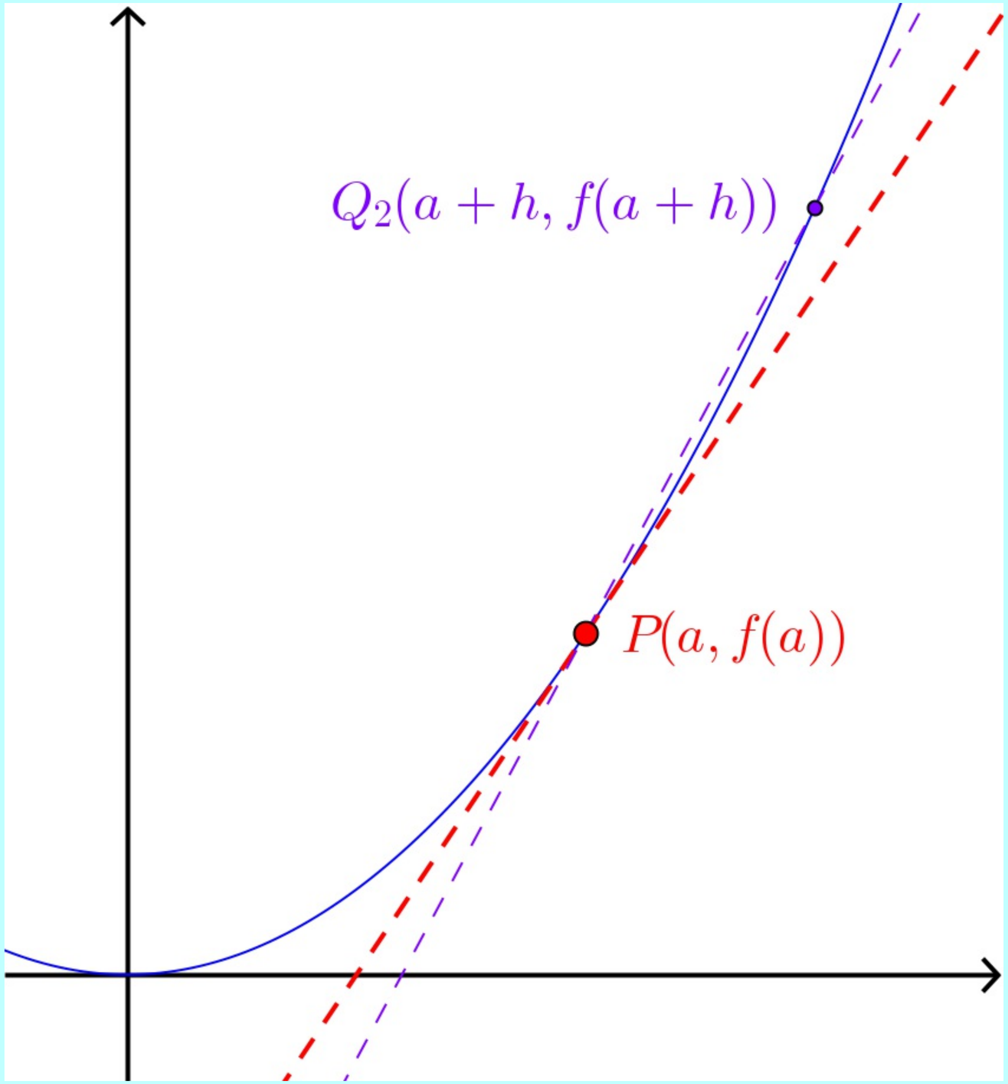
(b) a following interval uses a point after the point of interest.

(c) a centred interval uses points on either side of the point of interest.

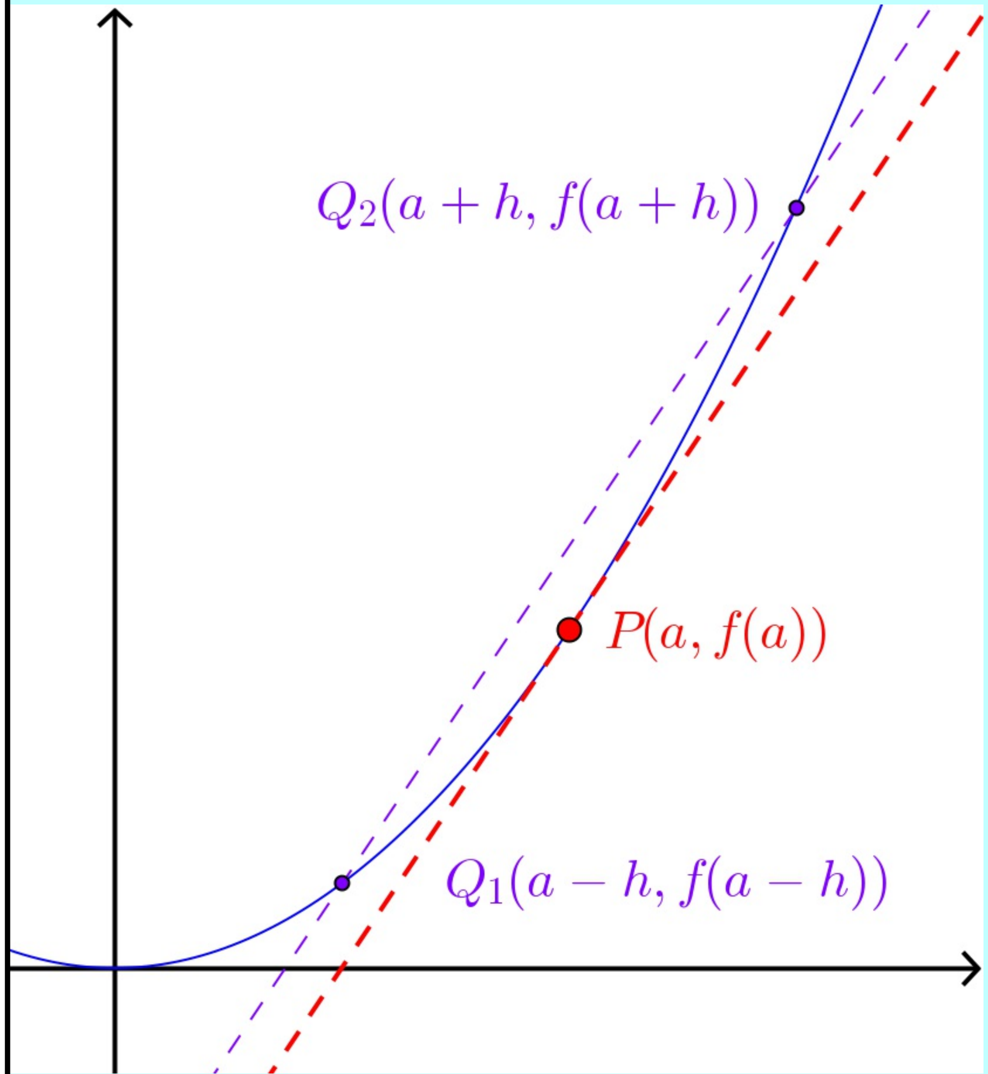
Preceding Interval



Following Interval



Centred Interval



Ex.1 A bacterial colony starts with 1000 bacteria and doubles each hour.

(a) Estimate the growth rate (bacteria/hour) after 2 hours (POI) using 1 hour intervals

(i) preceding

(ii) following

(iii) centred

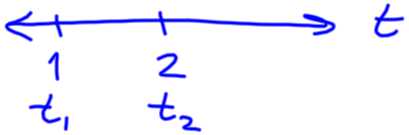
(b) improve the estimate using 0.1 hour intervals

$$P(t) = P_0(2)^{t/D}$$

$$\text{avg RoC} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$P(t) = 1000(2^t)$$

(a)



$$\text{avg RoC} = \frac{P(t_2) - P(t_1)}{t_2 - t_1}$$

$$= \frac{1000(2^2) - 1000(2^1)}{2 - 1}$$

$$= \frac{4000 - 2000}{1}$$

$$= \frac{2000}{1}$$

$$= 2000$$

In general, we algebraically represent the estimated instantaneous rate of change as a difference quotient.

For $x = a$, the point of interest is $P(a, f(a))$

The following point occurs at $x = a + h$, where h is an aribitrarily small value, giving a second point

$$Q(a+h, f(a+h))$$

$$\begin{aligned} \text{avg RoC} &= m_{PQ} \\ &= \frac{f(a+h)-f(a)}{(a+h)-a} \\ &= \frac{f(a+h)-f(a)}{h} \end{aligned}$$

To estimate instantaneous rate of change:

(a) Use a series of preceding and following intervals, keeping the point of interest constant. As the intervals get smaller and smaller, look for the trend in values.

(b) Use a series of centred intervals and look for the trend.

(c) Use the difference quotient for very small values of h (both positive and negative work).

The best estimates come from the smallest intervals.

In general, we will use a following interval and very small values of h.

Assigned Work:

p.76 # 8, 9

p.85 # 4, 7, 9, 10, 15

d b

p.76 #9 $h(t) = 18t - 0.8t^2$

$10 \le t \le 15$

t_1 t_2

$$\text{avg RoC} = \frac{h(t_2) - h(t_1)}{t_2 - t_1}$$

$$\text{avg RoC} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

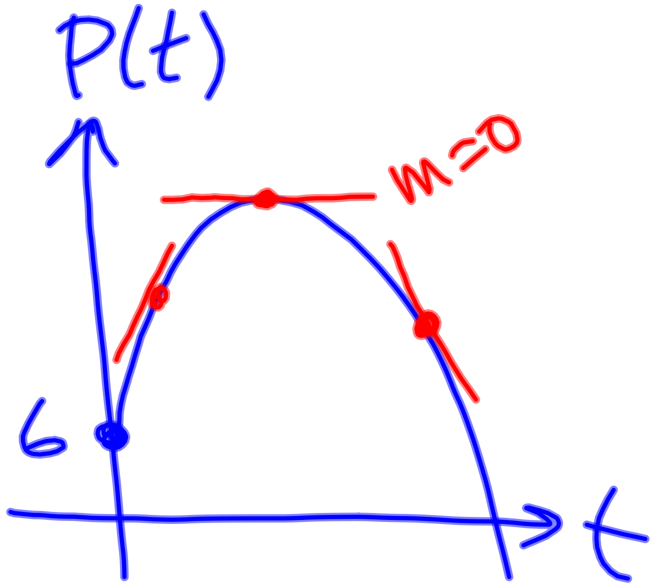
$$= \frac{h(15) - h(10)}{15 - 10}$$

$$= \frac{[18(15) - 0.8(15)^2] - [18(10) - 0.8(10)^2]}{5}$$

$$= -2$$

p. 85

#7.(d) $P(t) = -1.5t^2 + 36t + 6$



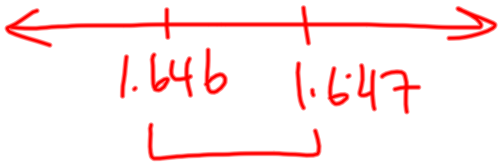
9. $h(t) = 10 + 2t - 4.9t^2$

(a) set $h(t) = 0$

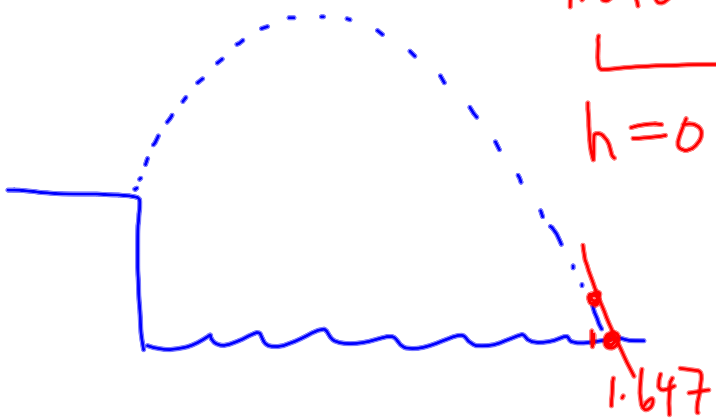
⋮

$t \doteq 1.647$

(b)



$h = 0.001$



$$iRoC \doteq \frac{h(1.647) - h(1.646)}{1.647 - 1.646}$$