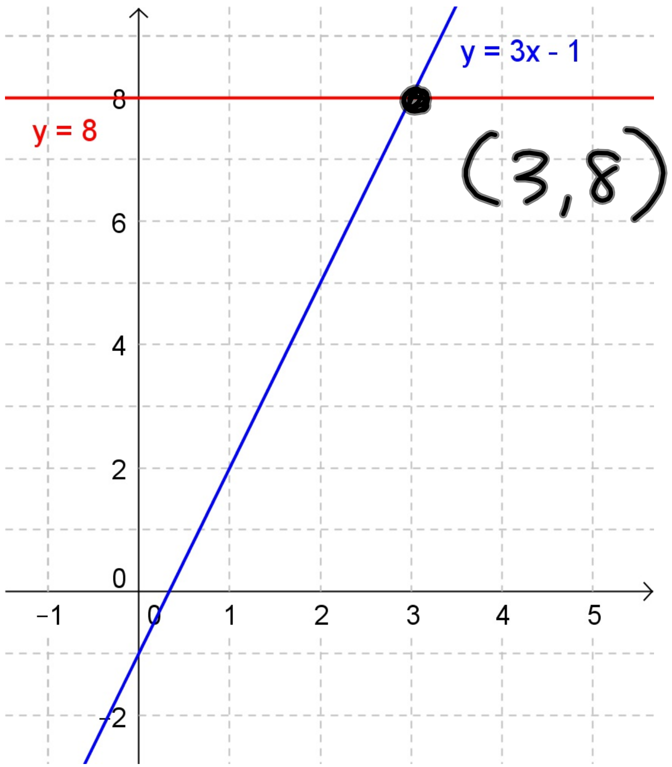
Solving Linear Inequalities

Oct 4/2016

To solve an inequality, find all values that satisfy the inequality.

Consider: $3x - 1 < 8$

The simplest way to visualize the solution is to graph and compare the LS and RS:



Where is the line y = 3x - 1 less than the line y = 8?

$x < 3$

We have also solved such inequalities by:

(1) solving the corresponding equation, then

(2) testing values around the solution(s).

(1) Solve $3x - 1 = 8$

$$3x = 9$$

$$x = 3$$

(2) Test x 3 < ___ and x > 3

$$3x - 1 < 8$$

test $x = 2$

$$LS = 3x - 1$$

$$= 3(2) - 1$$

$$= 5$$

$$RS = 8$$

$LS < RS$ ✓

test $x = 4$

$$LS = 3(4) - 1$$

$$= 12 - 1$$

$$= 11$$

$$RS = 8$$

$LS < RS$ ✗

$$3x - 1 < 8$$

(1) Solve
$$\begin{aligned} 3x - 1 &= 8 \\ 3x &= 9 \\ x &= 3 \end{aligned}$$

(2) Test $x < 3$: $3(2) - 1 = 5$, pass

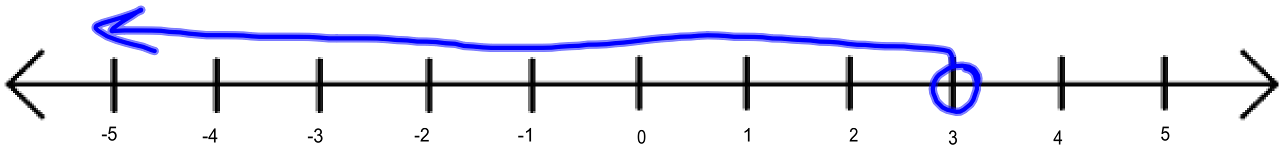
Test $x > 3$: $3(4) - 1 = 11$, fail

The solution can be represented as:

(a) set notation: $\{x \in \mathbb{R} | x < 3\}$

(b) interval notation: $x \in (-\infty, 3)$

(c) a number line:



Algebraic Operations on Inequalities

What are the effects of adding, subtracting, multiplying, and dividing on a very simple inequality?

Start with $4 < 8$, which is obviously true.

add positive:

$6 < 10$ ✓

add negative:

$3 < 7$ ✓

subtract positive:

$0 < 4$ ✓

subtract negative:

$14 < 18$ ✓

multply by positive:

$8 < 16$ ✓

multiply by negative:

$-4 < -8$

$-4 > -8$

divide by positive:

$1 < 2$ ✓

divide by negative:

$-1 < -2$

$-1 > -2$

Solving Inequalities Algebraically:

We can use the same basic operations (add, subtract, multiply, divide) that we would with a regular equation.

Note: When multiplying or dividing by a negative value, the direction of the inequality must be switched.

$[x \quad -3]$

Ex. Solve

(a) $2x - 3 > 5$

$$\frac{2x}{2} > \frac{8}{2}$$

$$x > 4$$

(b) $\frac{-1}{3}(x+4) \leq -7$

$$(x+4) \geq 21$$

$$x+4 \geq 21$$

$$x \geq 17$$

For a double-inequality, perform each operation on all parts simultaneously.

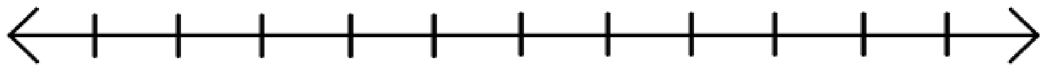
Ex. Solve $10 \leq 3(2x-5)-(3x-7) < 25$.

Express your solution using:

(a) set notation,

(b) interval notation,

(c) a number line.



$$10 \leq 6x - 15 - 3x + 7 < 25$$

$$10 \leq 3x - 8 < 25$$

$$+8 \quad\quad +8 \quad\quad +8$$

$$\frac{18}{3} \leq \frac{3x}{3} < \frac{33}{3}$$

$$\boxed{6 \leq x < 11}$$

Assigned Work:

p.213 # 5bdf, 6be, 7bdf, 8, 9, 11, 15, 19

b

(b) $-6x < x+4 < 12$

test $x = 0$:

LS	M	RS
$-6(0)$	$0+4$	12
$=0$	$=4$	

$\therefore LS < M < RS$

$\therefore$ 0 is a solution

8.

$$x > 4$$

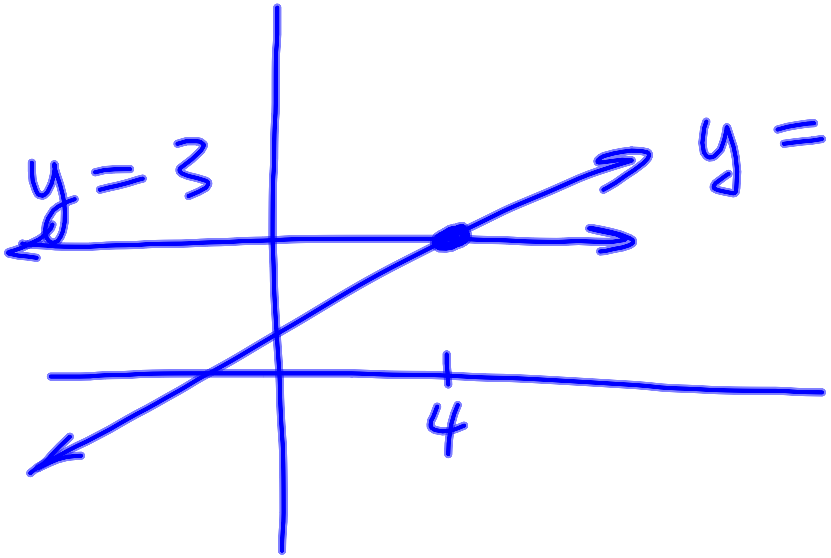
$$+x \qquad +x$$

$$2x > x + 4$$

$$-1 \qquad -1$$

$$2x - 1 > x + 3$$

11.



$\frac{1}{2}x+1 \le 3$ or $\frac{1}{2}x+1 > 3$

$\vdots$

$x \le 4$

19. (d) $\dfrac{-3x^3}{-3} \geq \dfrac{81}{-3}$

$$x^3 \leq -27$$

$$x^3+27 \leq 0$$

$$(x+3)(x^2-3x+9) \leq 0$$